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XXIII. *A Simple Method of Reducing Prismatic Spectra.* *By* EDWIN EDSER, *A.R.C.S., and* C. P. BUTLER, *A.R.C.S.*[*]

IN order to determine, from spectroscopic measurements, the wave-lengths corresponding to the bright lines in a prismatic spectrum of a metal or gas, one or other of the following methods is generally used. From preliminary measurements made of the deviation corresponding to a number of known lines in the solar spectrum, or the line spectrum of some metal or gas, a curve is drawn giving the relation between deviation and wave-length. Owing to the necessity of determining a very large number of points on this curve in order to render its form trustworthy, this operation is a very tedious one, and to an observer insufficiently acquainted with the reference spectrum involves great difficulty and uncertainty. This curve, however, having been drawn, the wave-length of any line in another spectrum obtained with the same spectrometer (no alteration of the adjustments having been made) can be immediately determined from a measurement of its deviation. On the other hand, where photographs of spectra are employed the most usual practice at present is to photograph a reference solar-spectrum alongside the one under examination. To an observer of sufficient experience it is possible to identify any of the numerous Fraunhofer lines with the corresponding lines in a Rowland's standard map ; and thus the wave-length of any line in the unknown spectrum may be determined by inspection. In spite of the perfection attainable by the above methods when employed by a trained observer, it has appeared to us that a simpler one, capable of giving accurate results in the hands of an experimenter without special experience in spectroscopy, might often be found of some value. The production of interference-bands in a continuous spectrum seemed capable of furnishing a reference spectrum which could be advantageously employed for this purpose, most of the difficulties incident to the above-mentioned methods being entirely eliminated. We have, therefore, devoted some time to the examination of various methods by which such interference-bands might be produced, with the object of selecting the simplest, and determining the degree of accuracy finally attainable by its employment. The results of our work in this direction we beg to lay before the Society this evening.

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[*] Read May 27, 1898.

If the image of a system of rectilinear interference-fringes be formed in the plane of, and parallel to, the collimator-slit of a spectrometer so that only a small part of the breadth of one band falls on the slit, the resulting spectrum will be crossed by vertical black bands, varying in number and breadth with the order of the interference-band from which the spectrum is derived*. If the interference-fringes are displaced across the slit the black bands in the spectrum will become finer and more numerous as the central interference-fringe recedes from the slit. Even when the coloured fringes have become invisible at the position of the slit owing to the high relative retardation of the interfering pencils, the bands in the spectrum remain quite distinct, becoming indistinguishable only when so fine that the resolving and dispersive powers of the spectroscope are insufficient to separate them.

In our earlier experiments we focussed the image of an air-film, contained between two plane and parallel glass surfaces, on the slit of the spectrometer †. The image should be obtained by means of light reflected from the film, the spectrum bands obtained when transmitted light is used being very faint. This method, which is theoretically the most perfect, has the disadvantage that a somewhat careful adjustment is necessary in order to insure good results. We have therefore sought for some simpler method. It is unnecessary here to detail the various methods which we have successively tried; it will suffice to describe the arrangement ultimately adopted as being the simplest, whilst complying sufficiently closely with the ideal conditions to insure trustworthy results.

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* Fizeau and Foucault, *Ann. de Chim. et de Phys.* 3rd series, tom. xxvi. p. 138 (1849); *Comptes Rendus*, Nov. 24, 1845.

† An air-film has been used somewhat similarly by Rubens in order to calibrate a prism for infra-red light. Wied. *Ann.* vol. xlv. (1892) p. 238.

Let us suppose that a transparent parallel-sided film of thickness d is placed immediately against the slit of the spectroscope and illumined with white light. Owing to the interference of the ray directly transmitted and that twice internally reflected within the film, there will be bright bands in the spectrum separated by darker intervals, the wave-lengths $\lambda_0, \lambda_1, \lambda_2, \ldots \lambda_r, \ldots \lambda_m$ corresponding to the bright bands being given by the equations

$$2\mu d = n\lambda_0 = (n+1)\lambda_1 = (n+2)\lambda_2 = \ldots = (n+r)\lambda_r = (n+m)\lambda_m,$$

where μ is the refractive index (supposed independent of the wave-length) of the substance of the film, whilst n may be any integral number.

If λ_0 and λ_m are known n can be determined from the equation

$$n = \frac{m\lambda_m}{\lambda_0 - \lambda_m}, \quad \ldots \ldots \quad (1)$$

where the interference-band at λ_m is the mth from that at λ_0; λ_m being towards the violet, and λ_0 towards the red end of the spectrum.

The wave-length λ_r corresponding to any other interference-band (the rth from that at λ_0) is now immediately given by

$$\lambda_r = \frac{n\lambda_0}{n+r}. \quad \ldots \ldots \quad (2)$$

It is therefore possible, by means of a series of interference-bands produced in the spectrum in the above manner, to calculate the wave-length corresponding to any part of the spectrum, having given any two lines of known wave-lengths sufficiently remote from each other.

Of course it is impossible to obtain a solid film of any substance whose dispersion is sufficiently small to render the above reasoning even approximately correct. Recourse must then be had to an air-film between two transparent plates. Since the film can now no longer be placed immediately against the collimator slit, some indefiniteness of the interference-bands will result; but if the plate next to the slit is not more than 3 millim. in thickness no trouble will arise from this cause, at any rate with a spectrometer whose collimator-tube is more than a foot in length.

A very considerable improvement in the interference-bands thus produced may be effected by partially silvering the two surfaces enclosing the air-film. In the first place the contrast between the bright and dark bands is considerably enhanced; indeed, if both surfaces be silvered so as to reflect about 75 per cent. of the incident light, the dark spectrum bands become almost black. The thicker the silver is the greater will be the contrast, the only limit being prescribed by the diminution of the total light transmitted.

Another important advantage gained by silvering the surfaces is the much sharper definition of the resulting bands. Messieurs C. Fabry and A. Perot* have pointed out that when monochromatic light is transmitted through a film enclosed between two plane and approximately parallel silvered surfaces, the resulting interference-bands present the appearance of sharp well-defined bright lines separated by broad black intervals. The explanation of this interesting phenomenon is quite simple. Let the real part of $\epsilon^{\iota p(x-Vt)}$ be the equation of the incident wave, whilst a and b are the respective coefficients of reflexion and transmission at the silver surface. Since Wiener has shown that the phase change for light reflected normally at a silver surface in air is very approximately equal to half a wave, a may be taken as wholly real.

Consequently, if the thickness of the air-film is d, the resultant transmitted beam on emergence will be given by the real part of the sum of the infinite series

$$b^2\epsilon^{\iota p\ -Vt+d)} + b^2a^2\epsilon^{\iota p(x-Vt+3d)} + b^2a^4\epsilon^{\iota p(x-Vt+5d)} + \&c.$$

$$= b^2\epsilon^{\iota p(x-Vt+d)}\{1 + a^2\epsilon^{2\iota pd} + a^4\epsilon^{4\iota pd} + \&c.\}$$

$$= b^2\epsilon^{\iota p(x-Vt+d)} \cdot \frac{1}{1-a^2\epsilon^{2\iota pd}} \cdot \qquad \text{. . . . .} \qquad (3)$$

Substituting $\cos 2pd + \iota \sin 2pd$ for $\epsilon^{2\iota pd}$, and rationalizing the denominator of (3), we find the transmitted wave to be equal to the real part of

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* *Ann. Chim. Phys.* xii. pp. 459–501 (1897).

$$\mathrm{A}\epsilon^{\iota p(x-\mathrm{V}t+d+e)},$$

where

$$\mathrm{A}^2=\frac{b^2}{1+a^4-2a^2\cos 2pd}, \quad . \quad . \quad . \quad . \quad (4)$$

and

$$\tan pe=\frac{\sin 2pd}{1-a^2\cos 2pd}.$$

Now A^2 is proportional to the intensity of the transmitted light; hence as d is varied the intensity will vary from $\mathrm{I}_{(\text{max.})}=\frac{b^2}{(1-a^2)^2}$ to $\mathrm{I}_{(\text{min.})}=\frac{b^2}{(1+a^2)^2}$. Taking Michelson's expression for the visibility of interference bands, viz., $\frac{\mathrm{I}_{(\text{max.})}-\mathrm{I}_{(\text{min.})}}{\mathrm{I}_{(\text{max.})}+\mathrm{I}_{(\text{min.})}}$, we find that this becomes equal to $\frac{2a^2}{1+a^4}$, which will have a maximum value when $a=1$. Thus the visibility of the bands will increase with the reflecting power, and therefore with the thickness of the silver.

Also, from (4) we obtain

$$\frac{d\mathrm{I}}{dd}=c\frac{-4pa^2b^2\sin 2pd}{(1+a^4-2a^2\cos 2pd)^2}.$$

Consequently I varies much more rapidly as d is increased when $2pd$ is nearly equal to $2n\pi$ than it does when $2pd$ is nearly equal to $(2n+1)\pi$. The bright bands will therefore be very narrow and sharply defined, separated by broad intervals very nearly black *.

It may be useful here to describe in detail the exact method of procedure finally adopted. It has not been found necessary to use optically worked glass; good ordinary plate-glass gives perfect results. Sextant glasses have been recommended to us for this purpose. It is well to select two plates having

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* A similar result has been noticed in connexion with the interference of electrical waves. See "Electrical Interference Phenomena, somewhat analogous to Newton's Rings, but exhibited by waves passing along wires of which a part differs from the rest," by E. H. Barton, D.Sc. Proc. Roy. Soc. vol. liv. p. 85.

the most suitable surfaces. This can be done by placing one plate on another, the two adjacent surfaces having previously been cleaned with cotton-wool, and viewing the air-film between them by reflected light from a sodium flame. The bands formed when the plates have been gently pressed together should be nearly straight and each one at least 2 millim. or 3 millim. in breadth.

The two selected surfaces should now be silvered somewhat heavily. The milk-sugar process*, in which the silver is deposited very slowly, has been found to give good results.

A simple mechanical arrangement for adjusting the two silvered surfaces for parallelism, the distance between them being also capable of adjustment, could easily be designed. We have found, however, that if a little soft wax be placed round the edges of the plates a perfect adjustment can be obtained by simply pressing the plates together with the fingers. The photograph accompanying this paper was obtained using this arrangement. To adjust for parallelism, view a spot of light, or the filament of an incandescent electric lamp, through the silvered surfaces. A long train of images, due to multiple reflexions, will generally be visible. These images having been brought into coincidence, interference-bands will generally be seen on viewing a sodium flame through the silvered surfaces. These are adjusted, by pressure applied to the glass plates, to be as broad as possible. When the adjustment is nearly completed there is often some difficulty in seeing the bands, due to the fact that for a parallel air-film viewed normally the interference-bands are formed at an infinite distance in front of the film †. At this stage the bands should be viewed from as great a distance as possible. The perfection of the results finally obtained will depend greatly on the accuracy with which this adjustment is performed.

If the collimator-slit of the spectrometer be now illumined

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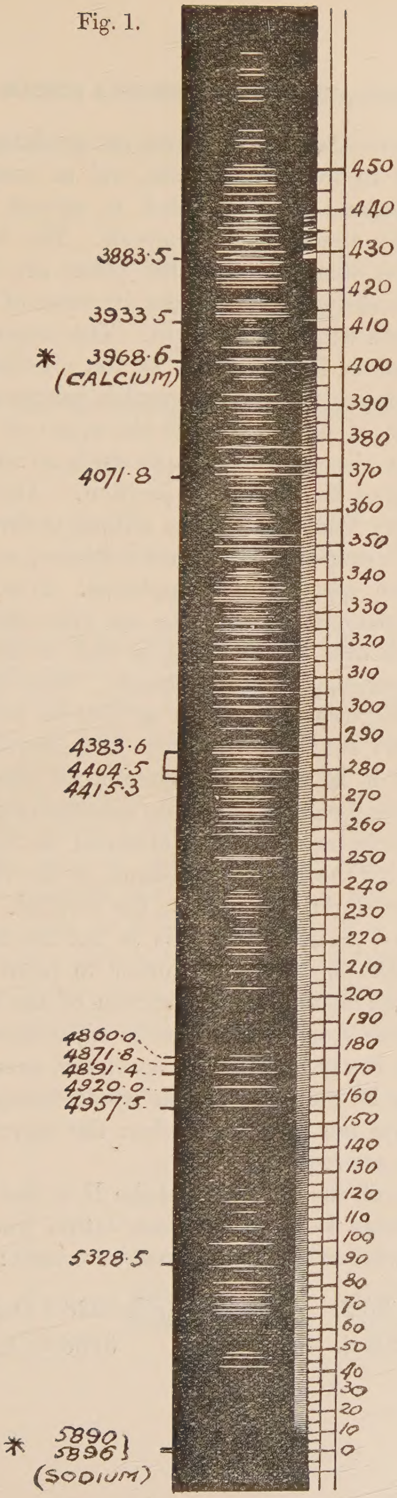
* For the exact process employed by us see 'Nature,' Sept. 23, 1897. "On the Phase-change of Light when Reflected from a Silver Surface," by Edwin Edser and H. Stansfield.

† A. A. Michelson on "Interference Phenomena in a new form of Refractometer," Phil. Mag., April 1882

Fig. 1.



COMPARISON SCALE FOR REDUCTION OF SPECTRA.

by a slightly convergent beam from an arc-lamp, and the plates be placed in front of the slit, and as near to it as possible, the spectrum will be found to consist of bright lines separated by almost black intervals. The best results will of course be obtained when the plates are in such a position that the slit is parallel to the direction of the interference-bands seen with sodium light. The closeness of the bands will depend on the thickness of the air-film between the silvered surfaces. For photographic purposes we have adopted the plan of covering either the upper or lower half of the slit with a piece of black paper stuck on with soft red wax before placing the plates in position. The necessary exposure will vary from about half a minute to three minutes (using Edwards's snap-shot isochromatic plates), according to the nature of the spectrometer employed. It is well to introduce a little common salt into the arc while this exposure is being made, as thus the D lines, as well as the H and K lines, will be superimposed on the bands. Another piece of black paper having been placed so as just to cover the exposed half of the slit, the first piece is removed, and the spectrum which it is wished to examine is photographed. Fig. 1 (p. 213) is a specimen of an iron spectrum together with a reference interference-spectrum obtained in this way. It will be noticed that the interference-bands in the violet part of the spectrum are slightly inclined to the vertical. It is easy to adjust the glass plates so that this is not the case, but we have selected this photograph in order to point out that if readings are taken on the line of junction of the two spectra no error will result from such a want of adjustment.

Starting from the red end of the spectrum, every fifth and tenth band can be marked and the whole numbered. The following exhibits the procedure when the wave-lengths of only a few lines are required.

One or other of the D lines and the H or the K line will generally be found to form the best datum lines. In the present case the following datum lines were used :—

Scale No. 90·2. Wave-length 5328·5 (λ_0)
" 402·3. " 3968·6 (λ_m) (H).

Then according to equation (1)

$$n=\frac{m\lambda_m}{\lambda_0-\lambda_m},$$

$$m=402{\cdot}3-90{\cdot}2=312{\cdot}1,$$

$$\lambda_0-\lambda_m=1359{\cdot}9\,;$$

hence

$$n=910{\cdot}8.$$

To find the wave-length of the line whose scale-number $=371{\cdot}2$:

$$r=371{\cdot}2-90{\cdot}2=281,$$

$$\lambda_r=\frac{n\lambda_0}{n+r}=\frac{910{\cdot}8\times5328{\cdot}5}{910{\cdot}8+281}$$

$$=4072{\cdot}2.$$

The true value of this wave-length is 4071·8, giving an error of +·4 tenth-metres.

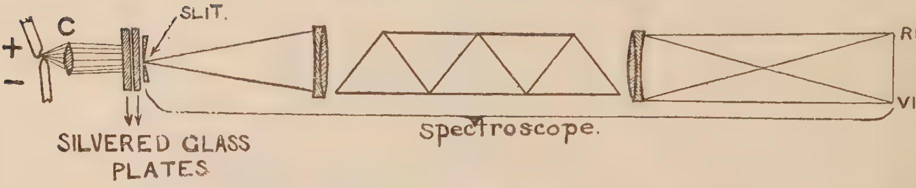
The following Table shows the calculated and true values for a number of lines in the above spectrum. It is given in order to indicate the degree of accuracy attainable. It is worth notice that these results were all obtained without the use of a travelling microscope, or in fact any auxiliary appliance other than an ordinary pocket-lens. With the latter it is easy to estimate the position of a line relatively to the interference-scale to within one-tenth of a band. Further, the interference-scale in the present instance was purposely made rather coarse so as to admit of reproduction. With a finer scale a greater degree of accuracy might be attained.

Scale No.	Wave-length (calculated).	True Wave-length.	Error.
			tenth-metres.
371·2	4072·2	4071·8	+·4
286·5	4383·7	4383·6	+·1
281·1	4405·2	4404·8	+·4
278·6	4415·2	4415·3	−·1
354	4131·8	4132·2	+·4
413·1	3933·9	3933·5 (K)	+·4

When it is required to determine the wave-length corresponding to a great number of spectral lines a graphical method may be employed. If we write $\frac{1}{\lambda}=\mathrm{L}=$ the frequency of the light vibrations, we obtain the simple relation $\frac{n+r}{\mathrm{L}}=$ constant, or $\mathrm{L}=\mathrm{K}(r+n)$, *i. e.* the relation between r and L may be expressed by a straight line.

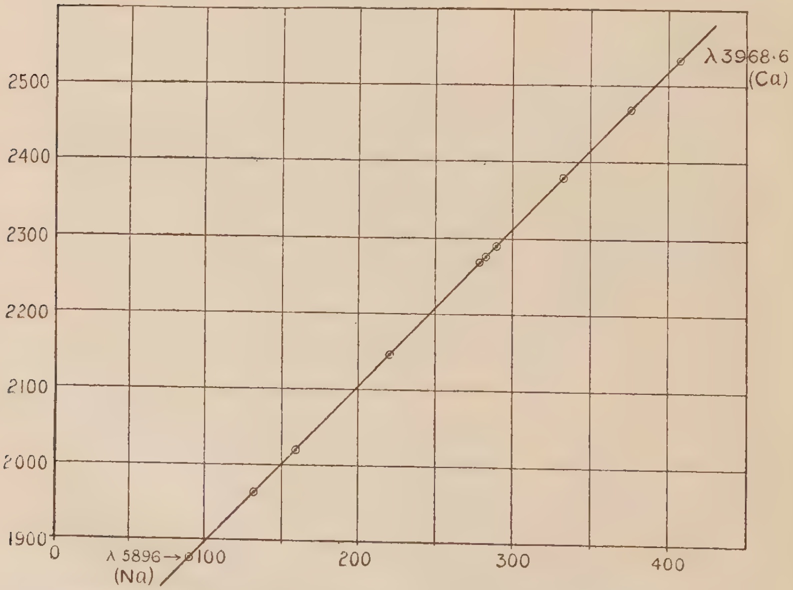
Fig. 2.

Diagrammatic Plan of Apparatus.



Plotting the frequencies vertically, and r horizontally, we obtain fig. 3. It is only necessary to mark off the scale-divisions, starting from zero, along the horizontal axis, and to mark off vertically above their respective scale-divisions

Fig. 3.



the frequencies of the two standard lines, joining the extremities of the latter by a straight line. The frequency corresponding to any given scale-number is read off directly. The accuracy attainable by this method, in which no calculations whatever are involved, is similar to that obtained by the method previously described.

It is, of course, unnecessary to take a separate photograph of the reference interference-scale for every spectrum to be examined. If the D lines are superposed on the original interference-scale, and occur also in every succeeding spectral photograph obtained, the reference scale can be photographed once for all, provided the adjustments of the spectrometer remain unaltered. The photographic scale can be placed with its film in contact with that of the photograph bearing the unknown spectrum, and the D lines having been brought into coincidence the procedure indicated above may be proceeded with.

For eye-observations the most convenient arrangement would be to place a small plate of optically worked glass between the reference prism, generally provided with a spectrometer, and the slit, a simple arrangement serving to adjust the adjacent surfaces (which should be silvered) for parallelism.

It will be seen that the phase-changes produced by the silver do not introduce any serious errors into the final results. Wiener* has shown that for light reflected from a silver film of sufficient thickness the phase-change is very nearly independent of the wave-length. To further test this a streak of silver was rubbed off the glass plate which is placed next to the collimator-slit, and a photograph of the spectral bands obtained. The displacement of the bands where the light had been reflected from the silver, relatively to the bands formed where the light had been reflected from the clear glass, was practically constant for the whole length of the spectrum.

In conclusion, we think that it may be claimed that by means of this application of a well-known principle to spectroscopy, the difficulties incident to the reduction of prismatic spectra in terms of wave-lengths or frequencies are greatly reduced, the whole process as above described requiring no special experience in the experimenter.

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* O. Wiener, Wied. *Ann.* xxxi. p. 629 (1887).

The experimental work incidental to this investigation has been performed partly at the Davy-Faraday Laboratory, Royal Institution, and partly at the Royal College of Science, South Kensington. For the facilities afforded us individually at these institutions our joint thanks are due.

Discussion.

Prof. THRELFALL congratulated the authors on their discovery of a method that would greatly reduce the labour of calibrating spectra, and at the same time give such accurate results.

Prof. BOYS said the simplicity of the apparatus added greatly to the value of the method. It would seem to him better if the slit were somehow contrived within the film-space. All want of definition due to rays falling at different angles upon the collimator object-glass would thus be avoided, and only a small part of the glass plates, *i.e.* the slit, would require to be strictly parallel planes. The limit of accuracy in the authors' method depended upon the collimator, not upon the optical perfection of the silvering of the plates.

Mr. BUTLER pointed out that previous methods had always required experienced spectroscopists for mapping out results. In the new method that work could easily be done by an assistant.

Mr. EDSER said that, by putting the two plates immediately in front of the slit, only a very small part of the glass is concerned in the action; light coming through at an angle would not reach the lens in the collimator.

XXIV. *Attenuation of Electric Waves along a Line of Negligible Leakage.* *By* Edwin H. Barton, *D.Sc., F.R.S.E., Senior Lecturer in Physics, University College, Nottingham*[*].

Shortly after the publication of my previous paper[†] on this subject, Mr. Oliver Heaviside pointed out to me that it would be interesting to compare the observed attenuation in the transit of the waves with that to be expected on the long-wave theory. He was also good enough to draw my attention to Lord Rayleigh's high-frequency formula for the effective resistance of wires to alternating currents as approximately applicable to the case. He feared, however, that the experimental value of the attenuation would be found considerably higher than the theoretical one due to resistance of the wires simply.

It immediately occurred to me that the extra attenuation observed over that theoretically expected might be due to the leakage conductance of the wood separators which were used at intervals along the line to keep the two parallel wires properly spaced. But at the time in question the line was already dismantled. It was therefore impossible to say precisely what the resistances of the separators were, nor how many of them were in use, when the attenuation was determined.

Assuming that the separators occurred on the average at a metre apart along the line, a resistance for each of the order half a megohm would suffice to reconcile the experiments with the theory referred to. On testing a few of the separators their resistance dry was found to be of the order ten megohms, but the resistance of one about a minute after momentary immersion in water had fallen to 0·2 of a megohm. It therefore seemed desirable to rig up a new line with separators of such high resistance as to render the leakage conductance of the line quite negligible. This was accordingly done and the attenuation again measured. The result, however, only confirmed the value previously obtained. This value is indeed higher than that predicted by the theory approximately applicable to the case. Still, upon careful examination of the differences between the experimental case and that contemplated in the theory, I think it will be conceded (1) that this difference of values is in the right direction, and (2) that it is not large enough to shake confidence in the substantial accuracy of the experiments. This is discussed in detail at the end of the paper.

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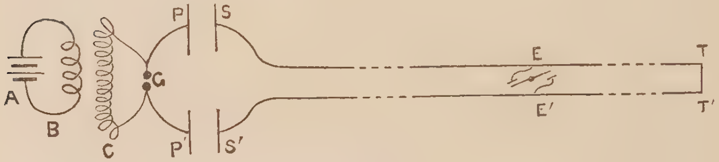
[*] Read June 10, 1898.

[†] Phil. Mag. August 1897; Proc. Phys. Soc. Dec. 1897, Jan. 1898.

Fig. 1.

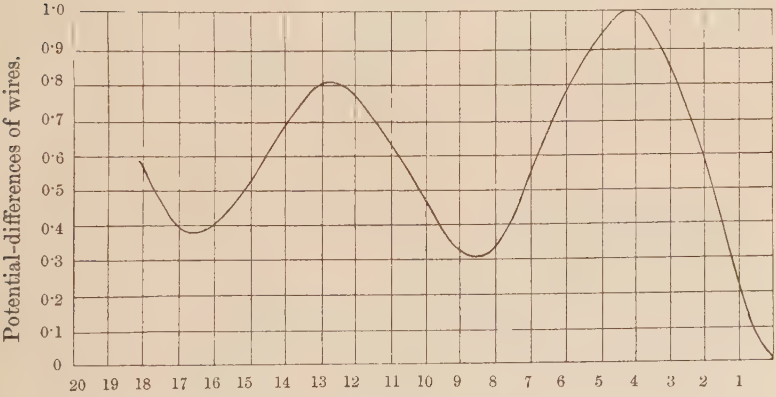


Experimental Arrangement.—The arrangement of the apparatus adopted in the determination is diagrammatically represented in fig. 1. In this figure, A denotes the battery of two storage-cells, B and C are the primary and secondary coils respectively of the induction-coil, of which C has a resistance of 3000 ohms and an inductance of about 20 henries. G is the spark-gap, which was adjusted to 2 millim. The wire PGP′, measured along the semicircle, was about 2 metres. PP′ are condenser-plates of zinc 40 centim. diam. placed opposite to and 30 centim. distant from the precisely similar plates S S′. The line is represented by S E T T′ E′ S′, and consists of two parallel copper wires 1·5 millim. diam., and kept 8 centim. apart by wood separators at intervals of about one metre. These wood separators are about 12 centim. long, 2·5 centim. wide, and 1·2 centim. thick, and were well saturated in melted paraffin-wax before use. Their resistance was thus raised to something over 60,000 megohms each, which rendered the leakage of the line quite negligible.

E E′ denotes the electrometer, which has a single plane needle, initially uncharged, and suspended by a fine quartz fibre between two disks attached to the line at E and E′. The needle is therefore electrified by induction whenever a wave passes E E′, and its ends are consequently attracted to the disks whatever the sign of their potential-difference. T T′ signify the two bridges alternately used at or near the end of the line, namely, (1) the bridge of critical resistance which absorbs the waves completely, and (2) the bridge which reflects them completely. The lengths of the line before and after the electrometer will be stated in connexion with each experiment. The waves generated by the primary oscillator were about 8·5 metres long, and, when on the line, are of the form represented in fig. 2.

Fig. 2.

Curve showing Wave-Train advancing to the right.



Lengths along wires in metres.

The electric waves propagated along the line are thus seen to be in the form of a damped train with the large end leading; the tail after about ten or a dozen waves being almost negligible. The curve of fig. 2 was obtained experimentally by Bjerknes' method, and is identical with that marked E in the paper on "Absorption of Electric Waves"*.

Method of Determination.—The method used for determining the attenuation of the waves in their transit along the line was the same as that described in the first paper already referred to. It consists essentially of alternately using at the end of the line a bridge which completely absorbs the waves and a bridge which completely reflects them. In the first case the wave-train goes but once past the electrometer, and gives a deflexion δ_1, say; in the second case the electrometer is affected by the sum to infinity of two geometrical progressions due to the forward and return trains respectively. Let the consequent deflexion be δ_2. Then the ratio r of δ_2 to δ_1 is a function of the attenuation factor $e^{-\sigma x}$, the reflexion coefficient at the oscillator, ρ, and the lengths of the line, l_1 and l_2, before and after the electrometer respectively. It is convenient, however, to solve the equations first for ρ^2 and s, where $10^{-sx'}=e^{-\sigma x}$, x' denoting lengths in metres and x lengths in centim. along the line.

Observations and Results.—The present determination is based upon observations with lengths of 116 metres and 65 metres respectively before the electrometer, and 48 metres in each case after. To determine the ratio δ_2/δ_1 in the second case 41 electrometer-throws were sufficient. These are given in Table I. In the first case, as the sparks of the oscillator were less regular, over a hundred electrometer-throws were taken before the required accuracy was obtained. Table II. summarizes the data and results.

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* Phil. Mag. January 1897, p. 43; Proc. Phys. Soc. vol. xv. p. 27

TABLE 1.

Showing electrometer readings for $l_1 = 65$ m., $l_2 = 48$ m.

Electrometer-throws with Absorbing Bridge only at end of Line.		Electrometer-throws with Reflecting Bridge		Ratios of Throws, viz. :—	
Actual observations.	Interpolated means.	at 48 m.$-\lambda/8$ beyond the Electrometer.	at 48 m.$+\lambda/8$ beyond the Electrometer.	$\frac{\text{col. 3}}{\text{col. 2}}$.	$\frac{\text{col. 4}}{\text{col. 2}}$.
20	21·5	66	…	3·07	
23	23	…	63	…	2·74
23	23·5	80	…	3·40	
24	24·5	…	77	…	3·14
25	25·5	80	…	3·14	
26	27	…	75	…	2·78
28	27	82	…	3·04	
26	24	…	72	…	3·00
22	24·5	79	…	3·22	
27	27	…	71	…	2·63
27	27	78	…	2·89	
27	26·5	…	72	…	2·72
26	27	81	…	3·00	
28	27	. .	77	…	2·85
26	26·5	76	…	2·87	
27	27	…	78	…	2·89
27	27·5	78	…	2·84	
28	26·5	…	76	…	2·87
25	26	79	…	3·04	
27	24·5	…	72	…	2·94
22					

Whence Final Mean Ratio = 2·955 ± 0·0254

Table II.

Summary of Data and Results of Experiments, with previous result added for comparison.

	Lengths of Line		Ratios of Throws $=r$.	The unknowns for which the equations were solved.		Attenuation Constant, σ.	Reflexion Coefficient at Oscillato ρ.
	before Electrometer $=l_1$.	after Electrometer $=l_2$.		s.	ρ^2.		
	metres.	metres.					
Present Determination.	116	48	2·555 ± 0·044	0·000566	0·513	0·0000130	0·716
	65	48	2·955 ± 0·0254				
Best of Previous Determinations.	117·5	20	2·41 ± 0·04	0·000564	0·4776	0·0000130	0·69
	65·0	20	2·744 ± 0·035				

Discussion of Results.—Heaviside has shown* that for electric waves along a pair of parallel leads the attenuation factor is

$$e^{-(\mathrm{R}'/2\mathrm{L}v+\mathrm{K}/2\mathrm{S}v)x}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where R', K, L, and S are respectively the effective resistance to the waves in use, the leakage conductance, the inductance, and the permittance, all per unit length of the line, v is the speed of light, x the space coordinate along the line, all units being on the C.G.S. electromagnetic system.

This, for the present case, reduces to

$$e^{-\mathrm{R}'x/2\mathrm{L}v}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

since $\mathrm{K}/2\mathrm{S}v$ is of the order one hundred-thousandth of $\mathrm{R}'/2\mathrm{L}v$.

Now to obtain an approximate value of R', use Lord Rayleigh's high-frequency formula †

$$\mathrm{R}'=\mathrm{R}\sqrt{\tfrac{1}{2}pa\mu}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

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* 'Electrical Papers,' vol. ii. p. 148, and elsewhere.

† Phil. Mag. May 1886, p. 390, equation (26).

where R is the resistance to steady currents of a length l of the wire, $\alpha = l/\mathrm{R} =$ the conductivity per unit length of the wire, $p = 2\pi$ times the frequency of the waves, and μ is the magnetic permeability of the wires.

In the experiments under discussion the numerical data were as follows:—

$\mathrm{R} = 1{\cdot}1 \times 10^5$ C.G.S. units per centim. (ascertained by P.O. Box tests, 10 metres of wire being 0·11 ohm).
$p = 2\pi \times 35 \times 10^6$ per second (deduced from λ and v).
$\mu = 1$ for copper wire. (In the intense field between the poles of an electromagnet the wire in question was found to be distinctly paramagnetic, but so was a roll of paper; thus μ is taken as unity.)

These values put in (3) yield

$$\mathrm{R}'/\mathrm{R} = 31{\cdot}6. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hence R for 1 centim. of the line, *i. e.* for 2 centim. of wire, is $2{\cdot}2 \times 10^5$; and we obtain

$$\mathrm{R}' \text{ per centim. of line} = 69{\cdot}5 \times 10^5. \quad . \quad . \quad (5)$$

Now for the line in question Lv has been determined *, both theoretically and experimentally, to be 56×10^{10} C.G.S. units.

Thus we have for the theoretical value of the attenuation constant

$$\sigma = \mathrm{R}'/2\mathrm{L}v = 0{\cdot}0000062, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

which is only half that determined experimentally.

In the endeavour to account for this discrepancy two lines of thought are open to us.

(1) We may examine the order of accuracy of the experiments and inquire in which direction the determination probably errs; and (2) the applicability to the case in point of the theoretical expression given may be discussed, and the sense of the modification which it requires for rigorous application to the present case may perhaps be inferred. These will be taken in the above order.

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* "Absorption of Electric Waves," Phil. Mag. Jan. 1897; Proc. Phys. Soc., Feb. and Mar. 1897.

The experimental value is calculated upon the supposition that the absorbing bridge used at the end of the line absorbs all the electric waves incident upon it, and that the reflecting bridge reflects all. If either supposition is incorrect the phenomena are changed, and the value found for the attenuation is in consequence affected.

Now it was previously found that the absorbing bridge for the line now in use must have a resistance of 560 ohms *. In the present work it was accordingly endeavoured to keep the resistance of the bridge at that value. It was adjusted before each occasion of its use and tested afterwards. The difference was usually of the order 2 or 3 ohms; but on one occasion it reached 10 ohms. However, to take an extreme case, let it be supposed that the critical value 560 is not the correct one, and that on some occasion of its use the bridge-resistance, though differing little from 560 ohms, differed by as much as 45 ohms from the value required for complete absorption of the electric waves incident upon it. Then the reflexion-coefficient of the bridge is given by †

$$\rho' = \frac{R_1 - Lv}{R_1 + Lv} = \frac{45}{2Lv + 45} = \frac{45}{1165}, \text{ nearly}, \quad . \quad . \quad . \quad (7)$$

where R_1 is the resistance of the bridge and Lv the instantaneous impedance of the line.

And it may easily be seen that the electrometer-throw when the absorbing bridge only is in use is, in consequence of its imperfect absorption, affected by a factor rather less than

$$(1 + \rho'^2)/(1 - \rho^2\rho'^2) = 1 + \tfrac{3}{2}\rho'^2, \text{ nearly}; \quad . \quad . \quad (8)$$

since ρ^2 is of the order one-half, ρ being the reflexion-coefficient of the oscillator.

Thus, for the supposed error of 45 ohms, which is an extreme case, we have $\rho' = 0{\cdot}04$ and the electrometer-throw increased in consequence by 0·24 per cent. But this is within the limits of the probable error of the determination of the ratios of the throws, and is therefore almost negligible.

Again, let us now examine the error consequent upon

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* "Absorption of Electric Waves," Phil. Mag. Jan. 1897; Proc. Phys. Soc. Feb. and Mar. 1897.

† Heaviside's 'Electrical Papers,' vol. ii. pp. 132–133.

incomplete reflexion from the bridge used to reflect all. The resistance of this bridge should be zero. It consists of a copper wire put across the line ; both it and the line were cleaned and polished at the places of contact, and the bridge made so as to spring on tightly into position. The bridge in question, put on with much less care than usual, showed a resistance of 0·04 ohm on testing with a P.O. box. But suppose by any mischance that the resistance of this bridge were half an ohm, then its reflexion-coefficient is given by

$$\rho'' = \frac{\mathrm{R}_2 - \mathrm{L}v}{\mathrm{R}_2 + \mathrm{L}v} = -\left(1 - \frac{2\mathrm{R}_2}{\mathrm{L}v}\right) = -(1-\chi), \text{ say}, \quad . \quad (9)$$

where R_2 is the resistance of the bridge and $\mathrm{L}v$ the instantaneous impedance of the line as before. Then the electrometer-throw when this bridge is in use is, in consequence of its imperfect reflexion, affected by a factor rather nearer unity than

$$\frac{1+\rho''^2}{2} \cdot \frac{1-\rho^2}{1-\rho^2\rho''^2} = \frac{1+(1-2\chi)}{2} \cdot \frac{1-\rho^2}{1-\rho^2(1-2\chi)} = 1-3\chi, \text{ nearly.} \quad (10)$$

And this factor, for half an ohm resistance, would diminish the electrometer-throw by 0·5 per cent. This again is almost negligible, being within the limits of the errors of determination of the ratios of the electrometer-throws.

Let us, however, in spite of the smallness of these effects on the electrometer-throw due to imperfect absorption or reflexion, inquire as to whether they are additive or compensatory. We at once see that they are additive, and that any departure from perfect absorption or perfect reflexion would result in a lessening of the observed ratio r of the electrometer-throws. Hence the true value of r which would be obtained under ideal conditions may be expected to exceed the actual one, if they differ at all.

In order to follow this possibility to its consequence, let us take each experimental value of r, *plus* its *probable error*, and from these data determine the corresponding value of σ. These probable errors, it will be noticed, exceed those which might be attributed to imperfection in the absorbing and reflecting arrangements, so entirely cover any errors which may be due to those sources. To still further test the consequences of extreme suppositions, values of the attenuation have been calculated for a value of one ratio r_1 *plus* its probable error, and the other r_2 *minus* its probable error, and *vice versâ.* The results are scheduled in Table III.

Table III.

The effect on the Attenuation of possible errors in the Ratios of the Electrometer-throws.

Ratios of electrometer-throws with length of line after electrometer 48 metres, and length before		Value of s where attenuation factor $=10^{-s}$ per metre.	Value of σ where attenuation factor $=e^{-\sigma}$ per centim.	Remarks.
116 metres.	65 metres.			
2·555	2·955	0·000566	0·0000130	Standard result.
2·599	2·980	0·000533	0·00001224	Result when each ratio is increased by its probable error.
2·599	2·930	0·000478	0·0000110	First ratio increased and second diminished by probable error.
2·511	2·980	0·000655	0·0000149	First ratio diminished and second increased by probable error.

It is thus seen that none of the suppositions considered bring the experimental value of the attenuation down to the theoretical value, though the slight difference made in the most probable case (second line of Table III.) is in the right direction, and lessens the value by nearly 6 per cent., whereas a reduction of 50 per cent. is required for agreement.

Turning now to the question of the validity of the expression for the effective resistance in the case under discussion: we have to notice (1) that this formula was developed for a wire whose return * was at a distance, and (2) that it is for "periodic currents following the harmonic law" †.

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* Phil. Mag. May 1886, p. 390.

† Ibid. p. 387.

Whereas in the actual experimental case we have

(1) The two wires comparatively near together (diameter 1·5 millim., distance of centres apart 8 centim.) ; and
(2) The wave-train is a rapidly damped one (see fig. 2).

To me it appears probable that each of these considerations if introduced into the theory would lead to a modification of the expression for the attenuation which would bring its value nearer the experimental one. Whether or not they would be competent to entirely bridge the discrepancy I do not pretend to say.

There are other possible causes of a discrepancy between the two values of σ. For example, any irregularity in the spacing or diameter of the wires would lead to a slight reflexion at the place of its occurrence, thus making the line behave towards the electric waves somewhat as a turbid or translucent medium behaves towards light. But as great care was taken in the spacing of the wires, no appreciable effect is likely to have arisen from this cause.

In conclusion I must thank Mr. Oliver Heaviside both for the suggestion which initiated this investigation and for his advice, ever freely given, during its continuance.

I am also specially indebted to one of our advanced students, Mr. Louis Lownds, for his able and painstaking assistance throughout the work both in the experimental and theoretical portions.

University College, Nottingham,
May 24, 1898.

XXV. *Diffusive Convection.*

By ALBERT GRIFFITHS, *M.Sc.* (*Vic.*), *A.R.C.S.* (*Lon.*). *

SECTION I.

THE diffusion of a substance through a liquid, *e. g.* water, may be regarded as merely the transmission of the substance through the stationary water.

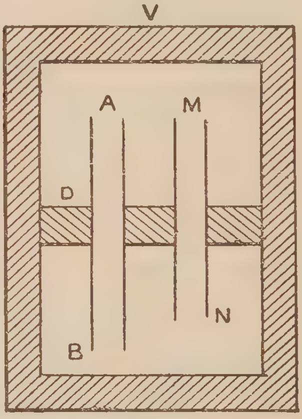
If the solution of the substance has a density different from that of water, depending on its strength, the act of diffusion tends to alter the density at any point: in general this tendency to alter the density prevents the possibility of statical equilibrium and hence convection-currents are produced.

So far as the author is aware the only case in which diffusion of the substance can take place without the production of convection-currents, is that in which diffusion takes place along a perfect cylinder the generating lines of which are truly vertical.

The determination of the magnitude of the convection-currents is, in general, a difficult task, involving, as it does, hydrodynamics as well as diffusion.

The present contribution determines the magnitude of the convection in a simple case, which arose in connexion with an attempt to devise an apparatus for the accurate determination of the coefficient of diffusion of copper sulphate through water.

Fig. 1.



Omitting some details, it may be said that the apparatus consists of a vessel, V, divided into two parts by a diaphragm D.

Through the diaphragm pass narrow tubes of which only two are shown in the figure. Means are taken for keeping the upper compartment full of nearly pure water, and the lower part full of a solution of copper sulphate.

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* Read June 10, 1898.

Attention will be confined to two tubes AB, MN, only, and it will be assumed that the upper ends A and M are in contact with perfectly pure water, and that the lower ends B and N are in contact with a solution of copper sulphate of *constant* strength. Doubtless diffusion will tend to diminish the strength of the solution at B and N; but there is reason to believe that the diminution in strength, under the conditions of the experiment, is inappreciable.

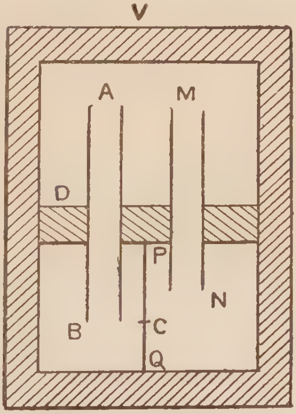
In the diagram A and M are shown at the same level, but AB is longer than MN, and B is at a lower level than N.

Under these circumstances it will be shown that, when things have attained a steady state, there is a flow of liquid *up* the tube AB, and down the tube MN.

To simplify the mathematical treatment of the problem, it will be assumed that the coefficient of diffusion is a constant, and that the density, d, of the copper sulphate solution is given by the equation $d=1+t$, where t is the quantity of copper sulphate per cubic centimetre of solution.

To show in a simple manner that the flow indicated must take place, imagine that the lower compartment is divided into two parts by a vertical partition, PQ, and that at C on a level with B there is placed a tap which can be opened or closed at will.

Fig. 2.



Let L_1=length of AB,
„ L_2= „ MN,
„ l =distance of a point in AB from the top A,
„ T =quantity of copper sulphate ($CuSO_4$) per cub. centim. in the lower compartment of the vessel V,
„ g =value of gravity.

When the tap at C is closed and the diffusion has attained the steady state, the density is given by the equation

$$d=1+t,$$

whence in the case of the tube AB,

$$d=1+\frac{l}{L_1}T.$$

The pressure at C on the *left*-hand side of the partition, neglecting the initial pressure at the upper ends of the tubes, is given by the equation

$$P_1=g\int_0^{L_1}\left(1+\frac{l}{L_1}T\right)dl$$
$$=gL_1\left(1+\frac{T}{2}\right).$$

The pressure on the *right*-hand side of C is given by the equation

$$P_2=gL_2\left(1+\frac{T}{2}\right)+g(L_1-L_2)(1+T).$$

P_2 is greater than P_1, the difference being equal to

$$\frac{gT(L_1-L_2)}{2}.$$

Since P_2 exceeds P_1, if the tap be opened liquid will flow from the right to the left.

If the tap be now closed, the diffusion will again attain a steady state, and the same inequality of pressure will be developed as before. The tap may again be opened and so on.

It is not necessary, however, to have a tap at C; if there is no obstacle whatever between B and N a steady flow will take place down MN and up AB.

The inequality between AB and MN produces, as we have seen, a pressure tending to produce circulation. On the assumption that the density is the same at all points of a horizontal layer, the magnitude of this pressure in dynes per square centim. equals

$$g\int_0^{L_2} d\,\partial l+g(L_1-L_2)(1+T)-g\int_0^{L_1} d\,\partial t,$$

where d is a function of the point under consideration in the tube AB or MN. To find the nature of this function the diffusion of copper sulphate, in a vertical tube, along which the liquid is moving, must be studied.

Section II.

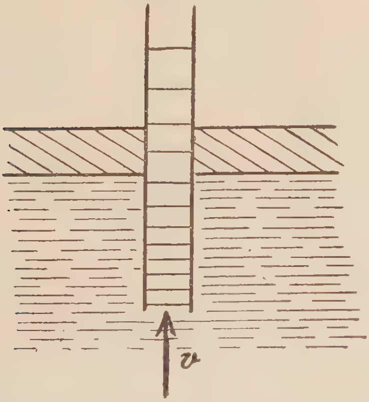
Diffusion of a salt in a vertical tube through which the liquid is flowing with a constant velocity, when the steady state has been attained.

Let l = distance of point from top of tube.
,, L = total length of tube.
,, A = sectional area of tube.
,, k = coefficient of diffusion.
,, t = quantity of dissolved salt per cub. centim. at a distance l from the top.
,, v = velocity of liquid up the tube.

When the steady state has been attained, the quantity of salt which passes in a second is a constant, and equals

$$vAt + Ak\frac{\partial t}{\partial l}.$$

Fig. 3.



If t is zero at the top of the tube, and T at the bottom of the tube, the solution of the differential equation which arises is

$$l = \frac{k}{v}\log_e \frac{T}{T - t\left(1 - e^{-\frac{vL}{k}}\right)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or

$$t = \mathrm{T}\frac{1-e^{-\frac{vl}{k}}}{1-e^{-\frac{v\mathrm{L}}{k}}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

By differentiating (1) and (2)

$$\frac{\partial t}{\partial l} = \frac{v}{k}\,\frac{\mathrm{T}-t\left(1-e^{-\frac{vl}{k}}\right)}{1-e^{-\frac{v\mathrm{L}}{k}}},$$

and substituting this value of $\frac{\partial t}{\partial l}$ in the expression $v\mathrm{A}t+\mathrm{A}k\frac{\partial t}{\partial l}$, we find for the *quantity* of substance which passes per second

$$\frac{v\mathrm{AT}}{1-e^{-\frac{v\mathrm{L}}{k}}}.$$

Let p=difference of pressure between bottom and top of tube, neglecting viscosity,

$$p = g\int_0^{\mathrm{L}} d\,\partial l$$

$$= g\int_0^{\mathrm{L}} (1+t)\partial l$$

$$= g\int_0^{\mathrm{L}} \left\{1+\mathrm{T}\,\frac{1-e^{-\frac{vl}{k}}}{1-e^{-\frac{v\mathrm{L}}{k}}}\right\}\partial l, \text{ from (2).}$$

Integrating

$$p = g\mathrm{L} + \frac{g\mathrm{TL}}{1-e^{-\frac{v\mathrm{L}}{k}}} - \frac{kg\mathrm{T}}{v}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

When the velocity is *downwards*

$$p = g\mathrm{L} + \frac{g\mathrm{TL}}{1-e^{\frac{v\mathrm{L}}{k}}} + \frac{kg\mathrm{T}}{v}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

SECTION III.

Determination of the magnitude of the convection-current in the case of tubes of equal bore, neglecting viscosity.

From (3) and (4), and from the expression in Section I., it can be seen that the pressure tending to produce circulation is given by the expression

$$\begin{aligned} P = gL_2 + \frac{gTL_2}{1-e^{\frac{vL_2}{k}}} + \frac{kgT}{v} + g(1+T)(L_1-L_2) \\ -gL_1 - \frac{gTL_1}{1-e^{-\frac{vL_1}{k}}} + \frac{kgT}{v}, \end{aligned}$$

or

$$P = gT(L_1-L_2) + \frac{2kgT}{v} + \frac{gTL_2}{1-e^{\frac{vL_2}{k}}} - \frac{gTL_1}{1-e^{-\frac{vL_1}{k}}}. \quad . \quad (5)$$

This pressure never can become negative in the positive direction; the limiting value of v is therefore given by the equation

$$gT(L_1-L_2) + \frac{2kgT}{v} + \frac{gTL_2}{1-e^{\frac{vL_2}{k}}} - \frac{gTL_1}{1-e^{-\frac{vL_1}{k}}} = 0. \quad . \quad . \quad (6)$$

In general the solution of this equation will involve considerable labour, but if $\frac{vL_1}{k}$ and $\frac{vL_2}{k}$ are small, it can be shown that

$$v = \frac{6k(L_1-L_2)}{L_1^2+L_2^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

or, if $L_1 - L_2 = \delta L$, and either length $= L$ approximately,

$$v = \frac{3k\delta L}{L^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

As a practical example take

$k = 2{\cdot}47 \times 10^{-6}$, the value of the coefficient of diffusion for copper sulphate,

$L = 4$ centim.,

$\delta L = 0{\cdot}05$ centim., *i.e.* half a millimetre.

Substituting in (8) we obtain

$v = 2{\cdot}306 \times 10^{-8}$ centim. per second

$= 5{\cdot}09$ centim. per year.

The author has tested the accuracy of this approximation in the original equation (6), and with seven figure logarithms has found the value of v to be exactly correct. He finds that (8) gives a value correct to a fraction of a per cent.

SECTION IV.

Effect of Diffusive Convection on the quantity of dissolved substance transmitted when the tubes are of equal bore.

The quantity transmitted, per second, of the tube L_1 equals from Section II.

$$\frac{vAT}{1-e^{-\frac{vL_1}{k}}};$$

expanding by the exponential theorem and dividing out we obtain

$$\frac{kAT}{L_1}\left(1+\frac{1}{2}\frac{vL_1}{k}+\frac{1}{12}\frac{v^2L_1^2}{k^2}+\&c.\right).$$

It is obvious that the expression within brackets is the correcting factor for the velocity of the liquid up the tube.

Taking the case considered in the previous section, with $L_1=4$ centim., and $v=2{\cdot}306\times10^{-8}$ centim. per second, the value of this correcting factor works out as 1·019 approximately; which shows that a velocity of about 5 centim. per year up a tube 4 centim. long produces an increase in the quantity of copper sulphate transmitted of about 2 per cent. This is a forcible illustration of the necessity for complete rest in diffusion experiments.

What is of chief importance here, however, is not the error produced in one tube, but the resultant error due to the flow in the two tubes.

The quantity transmitted along the two tubes is

$$\frac{vAT}{1-e^{-\frac{vL_1}{k}}}-\frac{vAT}{1-e^{\frac{vL_2}{k}}}$$

$$=\frac{kAT}{L_1}\left(1+\frac{1}{2}\frac{vL_1}{k}+\frac{1}{12}\frac{v^2L_1^2}{k^2}\right)+\frac{kAT}{L_2}\left(1-\frac{1}{2}\frac{vL_2}{k}+\frac{1}{12}\frac{v^2L_2^2}{k^2}\right)$$

$$=\frac{kAT}{L_1}\left(1+\frac{1}{12}\frac{v^2L_1^2}{k^2}\right)+\frac{kAT}{L_2}\left(1+\frac{1}{12}\frac{v^2L_2^2}{k^2}\right).$$

The correcting factor equals $1+\frac{1}{12}\frac{v^2L_1^2}{k^2}$ approximately; or, substituting the value obtained for v, $1+\frac{3}{4}\left(\frac{\delta L}{L_1}\right)^2$. Substituting the values given above for δL and L_1, the value obtained for the correcting factor is 1·00012 approximately; or the error due to the convection-current equals about one-hundredth of a per cent.

SECTION V.

Tubes of unequal bore.

Let $A =$ sectional area of the longer tube of length L_1 and
$r^2A =$ sectional area of the shorter tube of length L_2.
Let $v =$ velocity of flow in the shorter tube, then
$r^2v =$ velocity of flow in the longer tube.

By algebraical considerations similar to those already employed it can be shown that

$$v = \frac{6k\delta L}{L_2{}^2 + r^2L_1{}^2}$$

$$r^2v = \frac{6kr^2\delta L}{L_2{}^2 + r^2L_1{}^2}.$$

When $r^2 = 0$, the velocity in the shorter tube equals $\frac{6k\delta L}{L_2{}^2}$, whilst the velocity in the longer tube is zero.

When $r^2 = \infty$, the velocity in the shorter tube is zero, and the velocity in the longer tube is $\frac{6k\delta L}{L_1{}^2}$.

The maximum value of the velocity along a tube occurs when its bore is infinitely small in comparison with that of the other tube. In both cases the maximum value (since δL is small) equals $\frac{6k\delta L}{L_1{}^2}$ approximately; *i. e.* the maximum value is double the velocity produced when the tubes are of equal bore.

It can be shown by algebraical transformations that the correcting factor equals

$$1 + \frac{3(\delta L)^2 \times L_1L_2}{\left(\frac{L_2{}^2}{r} + rL_1{}^2\right)}.$$

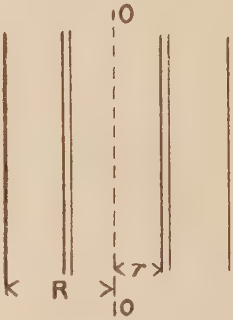
It is unity when r equals infinity or zero. Its *maximum* value occurs when $r = \frac{L_2}{L_1}$ (*i.e.* when the diameters of the tubes are in the same proportion as their lengths) and is $1+\frac{3}{4}\frac{(\delta L)^2}{L_1^{\,2}}$ approximately; it is seen that the maximum value is practically identical with the value in the case of tubes of equal bore.

SECTION VI.

Probable effect of Viscosity on the Convective Flow.

The internal friction of the moving liquid will diminish, to some extent, the velocity. It is obvious that the liquid in the interior portion of a tube will travel at a faster rate than that near the walls, and there is little doubt that the surfaces of equal density will not be horizontal planes. In all probability therefore the diffusion will take place radially as well as axially. It is also possible that the stream-lines will no longer be vertical; but superimposed on the vertical motion there may be slight eddies.

Fig. 4.



The author has not attempted the general problem, and in what follows only the ideal case is considered, in which both diffusion and motion take place in a vertical direction, *i. e.* their radial components are neglected.

Let OO′ represent the axis of the tube of radius R.

By considering the space between two concentric cylinders of radii r and δr, we obtain the equation

$$-\frac{\partial^2 v}{\partial r^2}\delta r\times 2\pi r \mathrm{L}\eta+\left(g\mathrm{L}+\frac{g\mathrm{TL}}{1-e^{-\frac{v\mathrm{L}}{k}}}-\frac{kg\mathrm{T}}{v}\right)\times 2\pi r\delta r$$

$$=p\times.2\pi r\delta r, \quad . \quad . \quad . \quad (9)$$

where v = the upward velocity,

η = the coefficient of viscosity,

p = difference between the pressures at the lower and upper ends of the tube.

Let $p=\left(1+\frac{\mathrm{T}}{2}\right)g\mathrm{L}+\mathrm{P}$, *i. e.* P is the excess of pressure above that necessary to produce statical equilibrium.

By algebraical transformation, neglecting the square and higher powers of $\frac{v}{k}$, equation (9) becomes

$$\frac{\partial^2 v}{\partial r^2}-\frac{1}{12}\frac{g\mathrm{TL}}{\eta k}v=\frac{-\mathrm{P}}{\mathrm{L}\eta}. \quad . \quad . \quad . \quad . \qquad (10)$$

Let

$$\frac{1}{12}\frac{g\mathrm{TL}}{\eta k}=f^2.$$

The solution of (10) is

$$v=\mathrm{M}_1e^{fr}+\mathrm{M}_2e^{-fr}+\frac{12\mathrm{P}k}{g\mathrm{TL}^2}. \quad . \quad . \quad . \quad . \qquad (11)$$

If there is no slipping at the walls of the tube $v=0$, when $r=\mathrm{R}$, hence $0=\mathrm{M}_1e^{f\mathrm{R}}+\mathrm{M}_2e^{-f\mathrm{R}}+\frac{12\mathrm{P}k}{g\mathrm{TL}^2}$; when

$$r=0,\ \frac{\partial v}{\partial r}=0, \text{ hence}$$

$$0=\mathrm{M}_1-\mathrm{M}_2.$$

Ultimately (11) becomes

$$v=\frac{12\mathrm{P}k}{g\mathrm{TL}^2}\left(1-\frac{e^{fr}+e^{-fr}}{e^{f\mathrm{R}}+e^{-f\mathrm{R}}}\right).$$

If $\eta=0$, it can be shown algebraically that

$$v=\frac{12\mathrm{P}k}{g\mathrm{TL}^2}.$$

What may be called the proportional-deviation equals

$$\frac{e^{fr}+e^{-fr}}{e^{f\mathrm{R}}+e^{-f\mathrm{R}}}.$$

To take a definite example let

$\mathrm{R}=0{\cdot}1$ centim.,
$\eta=0{\cdot}1324$,
$\mathrm{T}=0{\cdot}1$,
$k=2{\cdot}47\times10^{-6}$,
$\mathrm{L}=4$ centim.

Then $f=10{,}000$, and a study of the deviation shows that it is practically *nil*, except when r is nearly equal to R; in which case it equals

$$e^{f(r-\mathrm{R})} \text{ approximately.}$$

To make it equal to 0·001, r must equal 0·09931, and it is clear that, on the assumptions made, the effect of the viscosity is to produce variations in the average flow of only a fraction of a per cent. With tubes of ordinary calibre, no appreciable error is made if the viscosity be neglected altogether. The expressions also show that the tubes would have to be very narrow to affect the magnitude of the diffusive convection to any appreciable extent.

Another method of showing that very little error is made by neglecting the viscosity is to obtain an expression for the average velocity.

Let V = the average velocity up the tube, then

$$\mathrm{V} \times \pi\mathrm{R}^2 = \int_0^{\mathrm{R}} 2\pi r v \partial r$$

$$= \frac{12\mathrm{P}k}{g\mathrm{TL}^2} \frac{\left[\pi\mathrm{R}^2 - 2\pi\left\{\frac{\mathrm{R}}{f}(e^{f\mathrm{R}} - e^{-f\mathrm{R}}) - \frac{1}{f^2}(e^{f\mathrm{R}} + e^{-f\mathrm{R}}) + \frac{2}{f^2}\right\}\right]}{e^{f\mathrm{R}} + e^{-f\mathrm{R}}}.$$

Since f is large the second term within the square brackets may be neglected, and we obtain

$$\mathrm{V} = \frac{12\mathrm{P}k}{g\mathrm{TL}^2} \text{ approximately.}$$

Considering the case of two tubes of equal bore, but of unequal length, studied in Section III.,

let P_1 = "excess of pressure" producing the flow up the tube L_1.

P_2 = the corresponding excess (or defect) producing the flow down the tube L_2,

then it can be readily seen that

$$\mathrm{P}_2 = \frac{g\mathrm{T}}{2}(\mathrm{L}_1 - \mathrm{L}_2) - \mathrm{P}_1;$$

hence

$$\frac{g\mathrm{TL}_2{}^2\mathrm{V}_2}{12k} = \frac{g\mathrm{T}(\mathrm{L}_1 - \mathrm{L}_2)}{2} - \frac{g\mathrm{TL}_1{}^2\mathrm{V}}{12k}.$$

Putting $V_1 = V_2$, we obtain

$$V_1 = \frac{6k(L_1 - L_2)}{L_1^2 + L_2^2}.$$

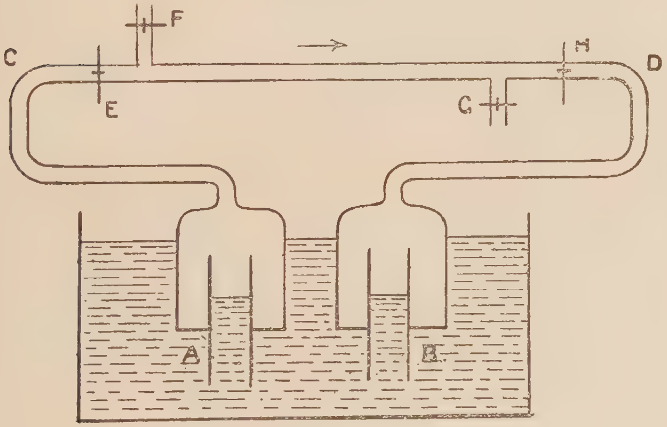
This is the same result as that obtained in Section III., when viscosity was neglected.

Section VII.

An Experimental Illustration of Diffusive Velocity.

The adjoining figure is a diagrammatic sketch of an apparatus kindly made by Mr. F. W. Rixon to test the existence of diffusive convection.

Fig. 5.



The upper extremities of two tubes, A and B, of equal diameter but slightly unequal in length, terminate in compartments connected by a capillary tube CD.

Any motion down B and up A is magnified in the proportion of the sectional area of A or B to the sectional area of the capillary tube.

The motion along CD is detected by the addition of copying-ink or other colouring matter. To introduce the ink, the taps E and H are closed, G and F opened, and the ink poured down F.

Owing to viscosity the velocity of the liquid at the centre is *double* the average velocity.

In the experiments, the results of which are recorded below, the tubes A, B, &c. were filled with water and placed in a strong solution of copper sulphate, the taps E, F, G, and H being closed. The taps E and H were then opened, and a rush would doubtless occur along CD. After some time, perhaps 5 or 10 minutes, the taps E and H were closed, and the ink added.

The chief data are as follows :—

Diameter of A or B = 1·09 centim. approximately.
Diameter of capillary = 0·019 centim. approximately.
Height of top of B above top of A = 0·08 centim.
Height of bottom of B above bottom of A = 0·292 centim.
Length of A = 4·622 centim.
Length of B = 4·410 centim.

The motion of the centre of the liquid in the capillary is indicated in the table below.

Hours from start ...	1	18½	43	66	90	114	162	189½	211	263	283	354.
Motion of centre in centim.	4·9	8·6	14·2	18·4	22·2	25·5	32·3	35·9	39·0	45·6	48·8	56·35.

If a curve be drawn with hours as abscissæ, and the distance moved as ordinates, a regular diminution of the flow is indicated, the velocity tending to a steady value. The average velocity in the last three days was 2·5 centim. per day.

Calculations made on the principles indicated in the previous section give about ·8 centim. per day as the velocity.

The observed velocity is thus three times the calculated. As the apparatus when originally put together was only intended to give qualitative results, it would be premature to discuss the cause of the difference *.

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* *Note added July 15th,* 1898.—Later experiments indicate that the difference between the calculated and observed velocities in the above was due to the continual increase of density of the solution owing to evaporation of water at the surface. On covering the surface with oil, to prevent evaporation, the observed and calculated values are found to be close enough to *prove the substantial truth of the theory.* Thus in one experiment, the observed motion of the index, at the end of a fortnight, was 2·3 centim. per day, whereas theory indicates that the final velocity should be 2·1 centim. per day.

After the above observations had been made, the tube B was lowered with respect to A ; the flow along the capillary from left to right was stopped, and there was probably a flow in the opposite direction. (The ink is not adapted for detecting slight backward motions.)

A second apparatus made by Mr. Rixon was fitted up, and a solution of common salt employed. Currents were produced in the direction indicated by theory. This new apparatus is capable of delicate adjustment, and experiments with it are still in progress. To the author the interest lies not so much in the confirmation of the theory as in the possibility of a physical method of determining the coefficient of diffusion.

A theoretical consideration suggested by the phenomena of diffusive convection may not be uninteresting.

As the water flows along the capillary, friction occurs and heat is produced. Ultimately the dissolved salt mingles completely with the water. Instead of allowing the flow to occur along the capillary the tap E or H might be closed. The dissolved salt would then mingle completely with the water without producing any flow, and the final state (assuming the apparatus is thermally insulated) is exactly the same as before.

Now the principle of the conservation of energy indicates that the heat produced in the capillary plus the heat developed by the mixing of the solution with water is a constant.

Does not this indicate that the heat produced on mixing a solution with water depends on how the mixing takes place? Is the matter connected with a sort of surface-tension existing in the spaces between a strong and weak solution?

XXVI. *Theory of the Hall Effect in a Binary Electrolyte.*

By F. G. DONNAN, *M.A., Ph.D.*[*]

IN 1883 Roiti[†] investigated the subject of a possible Hall effect in electrolytic solutions, but failed to obtain any positive result. Recently, however, the question has been taken up by Bagard[‡], who has obtained very considerable effects in aqueous solutions of zinc and copper sulphates. On the other hand, negative results have been obtained by Florio[§], and as both Bagard and Florio maintain the correctness of their experimental work, a polemic on the subject has arisen between them. In this condition of affairs it seemed worth while to examine what effect might be expected theoretically. With this purpose in view I made the calculation contained in the following pages. Subsequently I discovered that a similar theory had been given by Van Everdingen, jun.[‖]; but as I do not arrive at quite the same results, and have considered the subject somewhat more generally, it seemed to me not to be needless repetition to communicate this note.

The basis of the following calculation may be stated briefly as follows. The diagram shows the directions of the primary current and the magnetic field. The lines of flow of the primary current are supposed to be straight and the magnetic field everywhere uniform. The effect of the ponderomotive forces on the moving ions in the magnetic field is to urge both positive and negative ions in the positive direction of z. The moving ions will thus acquire component velocities in this direction, and these velocities being in general different for each sort of ion, the result is the separation of positive and negative charges, whereby a potential-gradient is set up in the direction of the z-axis which reduces the originally unequal velocities to equality. We thus obtain a flow of ionic matter in the positive z-direction. This produces in its turn a reverse osmotic gradient or concentration-fall both for dissociated and undissociated salt. The stationary state is finally attained when the net flux of ionic matter in the positive direction of z is balanced by the flux of undissociated salt in the opposite direction.

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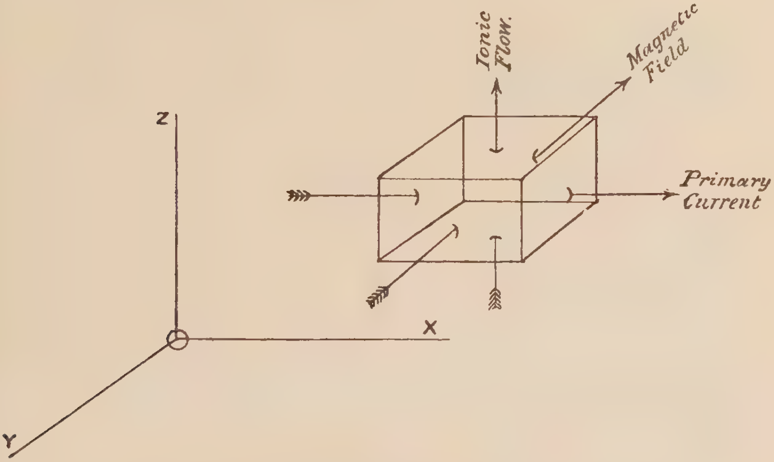
[*] Read June 24, 1898.

[†] *Journ. de Physique*, 1883.

[‡] *Comptes Rendus*, vol. cxxii. pp. 77–79, and cxxiii. pp. 1270–1273 (1896).

[§] *Nuovo Cimento*, [4] vol. iv. pp. 106–111 (1896).

[‖] *Metingen over het Verschijnsel van Hall en de Toename van den Weerstand in het Magnetisch Veld*, p. 102 *et seq.* (Leiden, 1897).



Let J = current-density of primary current,
c = concentration (in mols.) of positive *plus* negative ions,
p = corresponding osmotic pressure,
C = concentration (in mols.) of undissociated salt,
P = corresponding osmotic pressure,
u, v = velocities in centim. per second acquired under unit force by one gram-mol. of positive and negative ionic matter respectively,
G = velocity acquired under unit force by one gram-mol. of undissociated salt,
$\frac{d\pi}{dx}$ = potential-gradient of primary current,
$\frac{de}{dz}$ = potential-gradient of Hall-effect,
ϵ = quantity of electricity travelling per gram-equivalent of ionic matter,
ω = valency of each ion,
H = magnetic field-strength,
t = temperature (absolute).

We shall suppose the laws $p=c\mathrm{R}t$ and $\mathrm{P}=\mathrm{CR}t$ to hold for the solution in question, although this is not essential. We have furthermore

$$\mathrm{J}=-\frac{\omega^2\epsilon^2 c}{2}(u+v)\frac{d\pi}{dx}. \quad . \quad . \quad . \quad . \quad . \quad \text{(i.)}$$

Now an ion with a positive charge ωe moving with a velocity V in the positive direction of axis of x will be acted on by a force in the positive z direction equal to $\omega\epsilon\mathrm{VH}$; so that the force in this direction on a positive gram-ion is given by $-\omega^2\epsilon^2 u\mathrm{H}\frac{d\pi}{dx}$, whence it has a component velocity in the same direction amounting to $-\omega^2\epsilon^2 u^2\mathrm{H}\frac{d\pi}{dx}$. By similar reasoning the velocity of a negative gram-ion in the positive z direction is $-\omega^2\epsilon^2 v^2\mathrm{H}\frac{d\pi}{dx}$.

Expressing quantities of matter in mols., we can now draw up the following list, which takes into account all the fluxes of matter occurring in the solution.

1. Quantity of positive ionic matter traversing per unit time unit section perpendicular to axis of z in positive direction of this axis, due to ponderomotive forces arising from the magnetic field. $=-\frac{\omega^2\epsilon^2 c}{2}u^2\mathrm{H}\frac{d\pi}{dx}$.
2. Corresponding quantity of negative ionic matter. $=-\frac{\omega^2\epsilon^2 c}{2}v^2\mathrm{H}\frac{d\pi}{dx}$.
3. Flux of positive ionic matter in same direction due to potential-gradient along z-axis. $=-\frac{\omega\epsilon c}{2}u\frac{de}{dz}$.
4. Corresponding flux for negative ionic matter. $=+\frac{\omega\epsilon c}{2}v\frac{de}{dz}$.
5. Flux of positive ionic matter in same direction due to osmotic gradient. $=-u\frac{\mathrm{R}t}{2}\frac{dc}{dz}$.
6. Corresponding flux of negative ionic matter. $=-v\frac{\mathrm{R}t\,dc}{2\,dz}$.
7. Flux of undissociated salt in positive z direction due to osmotic gradient. $=-\mathrm{CR}t\frac{d\mathrm{C}}{dz}$.

The stationary condition then gives the equations:—

$$\frac{\omega^2\epsilon^2 c}{2} u^2 \mathrm{H} \frac{d\pi}{dx} + \frac{\omega\epsilon c}{2} u \frac{de}{dz} + \frac{u\mathrm{R}t}{2} \frac{dc}{dz} + \mathrm{GR}t \frac{d\mathrm{C}}{dz} = 0, \quad . \quad \text{(ii.)}$$

$$\frac{\omega^2\epsilon^2 c}{2} v^2 \mathrm{H} \frac{d\pi}{dx} - \frac{\omega\epsilon c}{2} v \frac{de}{dz} + \frac{v\mathrm{R}t}{2} \frac{dc}{dz} + \mathrm{GR}t \frac{d\mathrm{C}}{dz} = 0. \quad . \quad \text{(iii.)}$$

Elimination of $\frac{d\pi}{dx}$ and $\frac{d\mathrm{C}}{dz}$ from the equations (i.), (ii.), and (iii.) leads to the result

$$\frac{de}{dz} = \frac{2}{\omega\epsilon c} \frac{u-v}{u+v} \mathrm{HJ} - \frac{\mathrm{R}t}{\omega\epsilon c} \frac{u-v}{u+v} \cdot \frac{dc}{dz}. \quad . \quad . \quad . \quad \text{(iv.)}$$

It is to be observed that equation (iv.) holds for the variable as well as the stationary state, because although during the variable state the left-hand members of equations (ii.) and (iii.) are not zero, they are always equal to each other.

Accordingly for the initial phenomenon, before any appreciable concentration-gradient occurs, we obtain, putting $c = c_0$ and $\frac{dc}{dz} = 0$,

$$\frac{de}{dz} = \frac{2}{\omega\epsilon c_0} \frac{u-v}{u+v} \mathrm{HJ} = \omega\epsilon(v-u)\mathrm{H} \frac{d\pi}{dx}, \quad . \quad . \quad \text{(v.)}$$

or, integrating,

$$\iint \frac{de}{dz} dy\, dz = \frac{2\mathrm{H}}{\omega\epsilon c_0} \frac{u-v}{u+v} \iint \mathrm{J}\, dy\, dz\,;$$

whence

$$e = \frac{2}{\omega\epsilon c_0} \frac{u-v}{u+v} \frac{\mathrm{H}i}{d}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(vi.)}$$

where e = total difference of potential measured in direction of positive z-axis, d = thickness measured in direction of magnetic field, and i = primary current-strength. So that we obtain for the constant of the initial Hall-effect the value const. $= \frac{2}{\omega\epsilon c_0} \frac{u-v}{u+v}$.

In order to further investigate the stationary state, we shall suppose that the equilibrium-equation required by the laws of electrolytic dissociation is everywhere satisfied. This amounts to supposing that the processes which adjust this equilibrium proceed very much more rapidly than any of the other changes occurring in the system. In the uncertain state of knowledge concerning the equilibrium-equation it may be written for the present $C=\phi(c)$; so that $\frac{dC}{dz}=\phi'(c)\frac{dc}{dz}$. Eliminating $\frac{dC}{dz}$ between this equation and (ii.) and between this equation and (iii.), we obtain the equations

$$\frac{de}{dz}=\frac{2}{\omega\epsilon c}\frac{u}{u+v}\mathrm{HJ}-\frac{\mathrm{R}t}{\omega\epsilon c}\left\{\frac{2\mathrm{G}\phi'(c)}{u}+1\right\}\frac{dc}{dz}, \quad . \quad \text{(vii.)}$$

$$\frac{de}{dz}=-\frac{2}{\omega\epsilon c}\frac{v}{u+v}\mathrm{HJ}+\frac{\mathrm{R}t}{\omega\epsilon c}\left\{\frac{2\mathrm{G}\phi'(c)}{v}+1\right\}\frac{dc}{dz}. \quad . \quad \text{(viii.)}$$

Writing $\frac{2\mathrm{G}\phi'(c)}{u}+1=\mathrm{L}$, $\frac{2\mathrm{G}\phi'(c)}{v}+1=\mathrm{M}$, we obtain from (vii.) and (viii.)

$$\left.\begin{aligned}\frac{de}{dz}&=\frac{2}{\omega\epsilon c}\frac{1}{\mathrm{L}+\mathrm{M}}\frac{\mathrm{M}u-\mathrm{L}v}{u+v}\mathrm{HJ},\\ \text{or}\quad \frac{de}{dz}&=\frac{\omega\epsilon}{\mathrm{L}+\mathrm{M}}(\mathrm{L}v-\mathrm{M}u)\mathrm{H}\frac{d\pi}{dx}.\end{aligned}\right\} \quad . \quad . \quad . \quad \text{(ix.)}$$

For a completely dissociated electrolyte $\mathrm{L}=1$, $\mathrm{M}=1$, and we get instead of equations (ix.)

$$\frac{de}{dz}=\frac{1}{\omega\epsilon c}\frac{u-v}{u+v}\mathrm{HJ}=\frac{\omega\epsilon}{2}(v-u)\mathrm{H}\frac{d\pi}{dx}. \quad . \quad \text{(ix. }a\text{)}$$

Comparing equations (v.) and (ix. *a*), we see that the final potential-gradient for a completely dissociated electrolyte is just one-half the initial gradient.

Let $\frac{de}{dz}\Big/\frac{d\pi}{dx}=\mathrm{D}$, and denote by U, V the velocities acquired under unit potential gradient by a gram-mol. of positive and negative ionic matter respectively. Equation (ix. *a*) may then be written

$$\mathrm{D}=\tfrac{1}{2}(\mathrm{V}-\mathrm{U})\mathrm{H}. \quad . \quad . \quad . \quad . \quad . \quad \text{(x.)}$$

Van Everdingen * arrived at the equation $\mathrm{D}=(\mathrm{V}-\mathrm{U})\mathrm{H}$

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* *Loc. cit.* p. 207.

for the stationary state in an electrolyte supposed to be very slightly dissociated; but he assumes in this case that the ionic concentration does not vary throughout the solution, an assumption which is inconsistent with the equations (vii.) and (viii.), and therefore appears to me to be erroneous.

Equation (x.) may now be applied to the data obtained by Bagard. He finds, for example, the following results.

$CuSO_4$ solution. Concentration $=\frac{1}{16}$ grm. equiv. per litre. Temperature 21°–26° C.

H.	D.	$\frac{D}{H}$.
385 C.G.S. units.	·0018	$\cdot 46\times 10^{-5}$
707 ,, ,,	·0024	$\cdot 34\times 10^{-5}$
962 ,, ,,	·0034	$\cdot 35\times 10^{-5}$

To test these results by means of the theory, it is only necessary to calculate the value of $\frac{1}{2}(V-U)$, where it must be noticed that V and U are the velocities of the gram-mols. in centim. per second under a potential-gradient of one C.G.S. electromagnetic unit of potential per centim. From Ostwald's *Lehrbuch*, vol. ii. p. 770, the molecular conductivity of $CuSO_4$ solutions for the highest dilution at 18° C. is 217 in Siemens units, and therefore 230 in mho's. As the calculation is only very approximate, we may put consequently

$$(U+V)_{18^\circ}=\frac{230}{96540\times 2\times 10^8}.$$

From the same source, p. 612, we get $\frac{U}{U+V}=\cdot 36$, a value which may be regarded as fairly correct for temperatures in the neighbourhood of 20° C. (according to Hittorf and Bein, *loc. cit.*). Hence we obtain finally

$$\tfrac{1}{2}(V-U)_{18^\circ}=16\times 10^{-13}.$$

The value of $\frac{1}{2}(V-U)$ for temperatures 21°–26° C. will probably be somewhat smaller. Of course the solution of concentration $=\frac{1}{16}$ grm.-equiv. per litre is not by any means completely dissociated; and by using the proper equation, namely,

$$\frac{D}{H} = \omega\epsilon\,\frac{Lv - Mu}{L + M},$$

since $v > u$ and therefore $\frac{Lv - Mu}{L + M} > \frac{v - u}{2}$, we should obtain a higher value for $\frac{D}{H}$. Nevertheless this would never account for the difference between $\cdot 39 \times 10^{-5}$ as observed by Bagard and 16×10^{-13} as deduced theoretically for the case of complete dissociation.

So far as I can see, the theory here given is wholly in favour of the negative results obtained by Roiti and Florio; and it would therefore seem that Bagard has measured a phenomenon not contemplated in the foregoing theory. Van Everdingen in the paper referred to above supports Bagard; but this is owing to his having accidentally omitted the factor 10^{-8} in his numerical work.

The ionic concentration-fall can be readily calculated for a completely dissociated electrolyte. From equations (iv.) and (ix. *a*) we get at once

$$Rt\frac{dc}{dz} = HJ,$$

or

$$\frac{d \log c}{dz} = -\frac{\omega^2\epsilon^2}{2Rt}(u + v)H\frac{d\pi}{dx};$$

whence

$$\frac{1}{z_2 - z_1}\log\frac{c_1}{c_2} = \frac{\omega\epsilon(U + V)}{2Rt}H\frac{\pi_2 - \pi_1}{x_2 - x_1};$$

or

$$\frac{2Rt}{\omega\epsilon}\log\frac{c_1}{c_2} = H(U + V)(z_2 - z_1)\frac{\pi_2 - \pi_1}{x_2 - x_1}.$$

Thus, calling E the P.D. between two transverse electrodes (of the same metal as the kation) due only to the differences in concentration set up (a case which could be realized by taking an electrolyte such as silver nitrate, for which U and V are very nearly equal), it follows that

$$E = \tfrac{1}{2}H(U + V)(z_2 - z_1)\frac{\pi_2 - \pi_1}{x_2 - x_1}.$$

Taking the case of copper sulphate, we get, as before,

$$U+V=\cdot 00119.$$

Putting $z_2-z_1=1$ centim., and $\frac{\pi_2-\pi_1}{x_2-x_1}=1$ (volt per centim.), and $H=20,000$ C.G.S. units, we obtain

$$E=12\times 10^{-8} \text{ volts.}$$

Hence it would appear that in this, or in any other similar experimental arrangement, it would be necessary not only to employ an extremely strong magnetic field, but also a very high primary potential-gradient, *i. e.* of the order of 10,000 volts per centim. This might perhaps be realized experimentally by employing very dilute solutions and a large accumulator-battery, such as that recently employed by Professors Trowbridge and Richards *.

20th May, 1898.

Note added May 27th.—Dr. Van Everdingen informs me that he has discovered the slip in his calculation, and agrees with the remarks made above concerning the experiments of Florio and Bagard.

XXVII. *Apparatus illustrating the Action of Two Coupled Electric Motors.* *By* Prof. Carus Wilson.†

[Abstract.]

This is an apparatus to illustrate the action of two electric motors coupled in such a way as to admit of their rotating at different speeds. The two shafts are placed in line, and each fitted with a bevel wheel gearing into an intermediate wheel, the axis of which is at right angles to the line of the motor shafts, and free to rotate in a plane at right angles to that line.

The motors can be made to rotate at different speeds by altering the strength of the magnets of either or both. The motion of the intermediate wheel depends upon the difference of the two speeds if the two motors are rotating in opposite

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* Phil. Mag. February 1897.

† Read June 24, 1898.

directions, and upon their mean if they are rotating in the same direction.

A simple graphic construction enables us to predetermine the action of the arrangement with any given load on the intermediate wheel. Calling the two motors A and B, and the intermediate wheel C, we can draw lines representing, on a base of current, the speeds and the torques for each motor. If the motion of A and B are in the same direction, the load or torque will be the same on each and of similar sign. Hence as the load on the wheel C is increased, the speeds of A and B will tend to become equal, if A had been running faster than B, and for a certain load on C the speed of A will be equal to that of B. If the load on C is further increased, B will run quicker than A.

There will be a certain value for the load on C at which the motion of A will be reversed, and a further increase of the load on C will bring C to rest, A and B then rotating at equal speeds in opposite directions.

When the arrangement is such that the motors rotate in different directions when the load on C is nothing, suppose that A is running faster than B. The motion of C will depend upon the difference of the speeds of A and B. When a load is put on C, the motion of A is retarded, and that of B assisted, so that B takes less current and A takes more; the torques on the two motors, due to the load on C, now being of equal amount but of opposite sign. As the load on C is increased the speed of A will be reduced, and that of B increased until the two are equal and C comes to rest. B will now be acting as a generator, and sending a current on to A. If the load on C is simply that due to friction, the process cannot be carried any further. But if the load on C can be reversed, the speed of B will be greater than that of A, and the motion of C will be reversed.

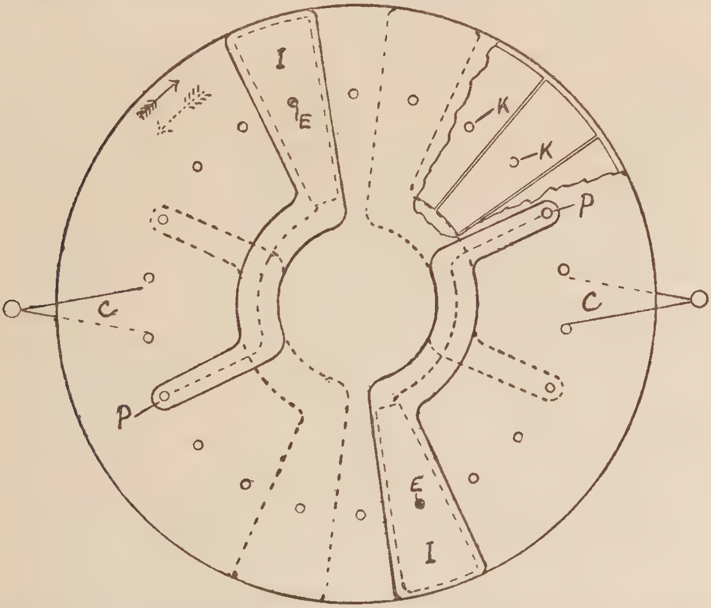
This arrangement has been made use of in an electric steering-gear designed by the Union Electricitäts Gesellschaft. The intermediate wheel is made to actuate the rudder by a differential action, and the motion reversed by making the speed of one motor greater or less than that of the other.

XXVIII. *An Influence-Machine.*

By W. R. Pidgeon, *M.A.*[*]

On June 23rd, 1893, I showed an influence-machine at a Meeting of this Society, which was described in the Phil. Mag. for September 1893, but which had the disadvantage of being an expensive machine to make. I now show a form of machine which not only gives better results, but is both cheap to manufacture, and has qualities which may, I think, interest the Members of this Society, especially in regard to its suitability for exciting Röntgen-ray tubes.

The machine consists of one or more pairs of glass disks mounted on a spindle and running in opposite directions, with earthing-brushes arranged similarly to a Wimshurst machine. The disks are of ordinary glass, and are covered with sectors about an inch or an inch and a half wide at the circumference, and placed about one-eighth of an inch apart.



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[*] Read October 28, 1898.

These sectors stand radially, and each carries a small brass contact-knob K. The disks are covered with wax composed of half paraffin and half rosin by weight. The wax covers up and insulates the whole of each sector except the small brass contact-knob which peeps above it. Each of the earthing-brushes, E, passes through and supports a fixed insulated inductor, I, which is formed of tinfoil stuck to an ebonite backing and insulated with wax. The surface of the wax on all the inductors and disks is carefully varnished several times with filtered shellac to protect the wax and give it a hard surface. Each inductor is kept charged by a stationary point, P, connected to it and placed so as to collect, from the revolving disk, shortly before the main collecting brushes C. The sectors on each of the disks are earthed at the moment when they are passing between the opposite disk on the one hand and the fixed inductor on the other, both of which carry a charge of the same sign. The sectors therefore receive their charge at a moment when their capacity is at a maximum, owing to their standing between two charged inductors. As each sector moves away from the brush to the right and out of the influence of the inductor its capacity decreases, and therefore its potential rises, and when it is opposite the point at which its fellow disk is being earthed, its potential is proportionately much higher than in the Wimshurst form of machine; and it therefore induces a proportionately higher charge on the sector being earthed. This sector, as it moves away from its inductor to the left, again rises in potential; and on arriving at the earthing-brush induces a still higher potential on the sectors moving to the right. This cumulative action goes on in a sort of geometrical progression until, as a matter of fact, the output of this form of machine rises to about four times that of a Wimshurst of a similar size when measured by the overflow of a leyden-jar (25 oz.).

It may help to make the action of the machine more clear if we regard it from the point of view of its being a condenser, the plates of which can be charged in one position, then shuffled, to bring the positives and negatives together, and thus discharged. For the fact that each sector is imbedded in an insulator enables it to receive a charge on each face as it stands between the disk and the inductor, like a plate in a condenser. It therefore carries forward a double charge, so to speak, as compared with that carried forward by a machine without inductors. Again, the drop of capacity and consequent rise in potential of the sector as it moves away from the inductor is so great, that the induction of the machine is also practically doubled, and hence the total output is multiplied by four. That is to say, a machine having inductors which act upon numerous insulated sectors is equivalent in output to four machines of the same size of the ordinary type.

W. R. Pidgeon's machine with one pair of plates—			
of 12 in. diam. requires	22 sq. ft. of area to pass the collecting-brushes per spark.	A 15-in. Wimshurst same day requires	64 sq. ft. per spark.
of 17 in. diam. do.	17·1 sq. ft. do.	A 15-in. do.	76 do. do.
18 in. do.	18·5 do.	A 15-in. do.	70 do. do.
19 in. do.	19·4 do.	A 15-in. do.	70 do. do.
2 pairs of 27 in. do.	16·4 do.		
Average ……	17·85 sq. ft.	Average…	70 sq. ft.

Comparative efficiency nearly 4 to 1.

Mr. Wimshurst's 8-plate 15-in. machine, which he kindly tried for me himself, requires 97·07 sq. ft. per spark.

The length of spark between knobs is approximately the same as in a Wimshurst of the same size, but, if anything, slightly less. If, however, the fixed inductors are taken away and the wax disks run with the brushes on alone, the machine gives much longer sparks; but its output is then decreased to a little less than double that of a Wimshurst.

My machine, 19 in., without inductors, requires 45 sq. ft. per spark.

The wax which covers the disks prevents the sectors leaking from any point except the small brass contact-knobs, and thus enables the machine to work in the dampest weather; in fact, it may be sponged with water, or have water squirted at it, and yet will work if only it is first wiped up with a duster. Dirt, likewise, makes almost no difference; and usually the induction starts up before the disks have made a revolution, even though the machine may have been left standing for weeks. It will, moreover, work on short circuit;

and in all but the very worst weather, or after standing idle for a long time, it is not necessary for any of the brushes to actually touch the contact-knobs.

The fact that the only place from which a sector can leak electrically is from its small brass collecting-knob, makes it possible to run disks so large as to almost touch the first motion-shaft below or the collectors on either hand. This obviously allows the machine to be snugged up, and so save cupboard-room.

The collectors have been covered with ebonite, and everything has been done to expose as little naked surface as possible, partly for convenience in handling, but chiefly to enable the machine to be used in bad weather.

When exciting a Crookes or Jackson tube the knobs of the dischargers should be brought to either end of the tube, the terminals of which should also be capped with brass knobs to prevent any brush-discharge.

The tube should be of a sufficiently high resistance to use all the potential of the machine and not require a spark-gap; it should be roughly suitable for a coil-spark of 6 in. to 14 in., according to the size of machine with which it is excited.

The illumination produced will then be good and steady, and the tube may be run for an almost indefinite period without running the slightest danger of over-heating its terminals or of being troubled by its resistance changing.

A pair of 19-in. disks is adequate to show brightly the bones in the hand and arm, and, with some people, to faintly indicate the ribs on a screen; while a pair of 12-in. disks exciting a suitable tube is sufficient to show the hand- and wrist-bones clearly.

Discussion.

Capt. J. H. Thomson, R.A., said that, apart from its electrical merits, the machine possessed advantages in mechanical construction. He thought there was still room for improvement in this respect. The counter-shaft should be done away with, and ball-bearings should be introduced. The inductor was a distinct improvement; he thought the efficiency might be increased by adding other inductors. Platinum-iridium was the best material for brushes of such machines.

Prof. AYRTON asked what efficiency was obtained with modern influence-machines in general.

Capt. THOMSON had found that when running a machine by a motor, about 80 per cent. of the power was wasted in mechanical friction ; of the remaining 20 per cent. a great deal was lost as electrical leakage.

Prof. S. P. THOMPSON thought it had been pointed out by Mr. Wimshurst that influence-machines did not work well unless there were at least two thicknesses of glass between the inducing and induced conductors. That was why Mr. Wimshurst put his sectors on the outer faces of the glass disks. Mr. Pidgeon had departed from this. The advantage of the narrow spacing of the sectors was not very apparent.

Mr. WIMSHURST (abstract of communication). Waxing the disks reduces leakage and increases the output ; the wax-coating virtually doubles the number of plates. Inductors contribute a further increase to the output. In 1883 Mr. Wimshurst tried thick coatings of shellac, and also duplicating the glass, with in some cases sectors upon the second glass to increase the capacity. The output was increased, but the construction lost simplicity. The indifference of Mr. Pidgeon's machine to dirt and dust was a most valuable result.

Mr. PIDGEON, in reply, showed a set of secondary inductors such as Capt. Thomson had just proposed. They improved the output by about 15 per cent., but they were troublesome to keep in order, for they increased the tendency to "reverse."

XXIX. *The Magnetic Fluxes in Meters and other Electrical Instruments.* *By* Albert Campbell, *B.A.*[*]

In all electrical measuring instruments in which the deflecting or controlling forces are electromagnetic, the magnetic fluxes and fields are of great importance, and yet there seem to be no tables published which give even rough measurements of these, the result being that many people who are thoroughly expert in the use of instruments have no idea whatever of the order of magnitude of the magnetic fluxes occurring in the very commonest instruments. In order, therefore, to fill this gap to some extent, I have recently carried out a series of experiments on the subject, and although the list of instruments thus tested is not very extensive or complete, I have been able to include in it a good many of the more familiar types. As individual instruments of the same type vary somewhat amongst themselves, it would have been waste of time to have aimed at great accuracy in these measurements. Accordingly, whilst guarding against large errors in general, I have been content in one or two cases with results which only indicate the order of magnitude of the quantity measured.

In most cases the quantity determined has been B, the magnetic flux density or number of induction-tubes per square centimetre sometimes through iron, sometimes through air. In some cases Φ, the total flux, was measured, and for several of the meters determinations of the power lost in their various parts were also made.

Methods of measuring B.

Method I. Except when the fluxes were alternating, the method used was the well-known way with a ballistic galvanometer. A search-coil in circuit with the galvanometer either had the flux suddenly reversed in it, or was pulled quickly out of the field. For many of the experiments small and very thin search-coils had to be used. For instance, in one of the meters, the available air-gap was only 1 millim. across. The bobbins of these small coils were made by cementing together one or more round microscope cover-glasses between larger strips of mica for the ends; they were wound with from 10 to 200 turns of silk-covered copper wire of 0·075 millim. diameter. Much thicker wire was used when it was desirable to have the resistance of the galvanometer-circuit low. The smoothness of the mica cheeks allowed the coils to be withdrawn from position with the necessary quickness. The galvanometer was calibrated from time to time by means of a standard pair of coils whose mutual inductance was accurately known. As the measurement of small fluxes by the ballistic method presents no difficulty, and requires only ordinary instruments, further description is needless.

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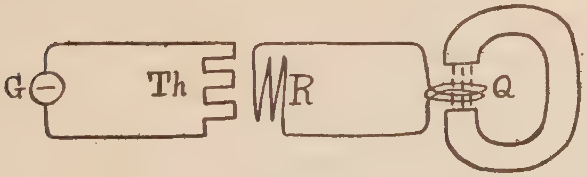
[*] Read October 28, 1898.

Method II. When, however, the small flux is an alternating one, the voltage set up in the search-coil is more difficult to measure; accordingly two special methods were here used. In the more accurate of these two methods the search-coil Q (fig. 1), through which the flux was made to pass, instead of being in circuit with a ballistic galvanometer, was joined directly to a resistance-coil R, laid along one set of junctions of a minute thermopile Th, which last was connected with a sensitive galvanometer G. The resistance-wire was of manganin (silk-covered) and was placed along the junctions so as to be non-inductive, and to avoid producing eddy-currents. The thermopile consisted of ten pairs of thin iron and nickel wires each 7 millim. long. These metals were chosen as their thermo-electric lines are far apart, and almost parallel to one another.

Fig. 1.



Some years ago the writer showed (Proc. Roy. Soc. Edin. July 1887) that a thermopile used thus could give a fairly accurate measurement of the current through the resistance-wire, the ultimate deflexion being proportional to the square of this current. Hence, for a given frequency, the mean square of the P.D. at the terminals of the search-coil was proportional to the deflexion. Each time it was used the combination was calibrated in the following manner:—A measured current of 1 ampere from the alternating supply circuit used was passed through a non-inductive resistance of 0·2 ohm, and the resulting P.D. of 0·2 volt was applied to the ends of the resistance R. From the observed deflexion of the galvanometer the mean square of the volts per division was found*. In all cases when the search-coil was in circuit the frequency n was observed, being measured by a frequency-teller. If the resistance of the search-coil be negligible, and if the flux follows the sine law, we have

$$v_2 = 10^{-8} \times 2\pi n N_2 B s_2,$$

where

v_2 = voltage shown by galvanometer,
n = frequency,
N_2 = number of turns in search-coil,
s_2 = area of search-coil,

and

$$B = \sqrt{\text{mean square B}}.$$

The other method, which was by means of a telephone, was a rougher way, and will be described below.

In Table I. are given some of the results obtained in the case of the simpler instruments, the third column giving the resistance of the instrument, the fourth column its full load, and the fifth the mean value of B at that load. As far as possible the positions from which the mean value of B were obtained were chosen so as to give an idea of the *working* flux-density, and except where otherwise stated the values given refer to full load.

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* The thermopile method of measuring small voltages is now in use in the German Reichsanstalt.

TABLE I.

No.	Name.	Resistance, Ohms.	Full Load.	B (mean).
1.	Siemens Electrodynamometer......	0·58	4 amps.	80
	„ „	0·0156	20 amps.	18
2.	Kelvin Ampere-Balance	0·58	10 amps.	65
3.	Bifilar Mirror Wattmeter (after Dr. Fleming).	0·0078	50 amps.	55
4.	Ayrton and Mather D'Arsonval Galvanometer.	324	......	450
5.	Weston Voltmeter	7·4	0·2 volt	870
6.	Davies Voltmeter (Muirhead) ...	481	3 volts	400
7.	Evershed Ammeter	0·00043	100 amps.	700
8.	Ayrton and Perry Magnifying Spring Voltmeter.	30	18 volts	14,200
9.	Richard Recording Ammeter	7×10^{-5}	500 amps.	580
10.	Dolivo Voltmeter	56	15 volts	75
11.	Nalder Voltmeter......................	586	40 volts	70
12.	Any Tangent Galvanometer	......	Defl.=45°	0·26
13.	Kelvin Astatic Mirror Galvanometer.	13,000	......	0·008
14.	Evershed Ohmmeter, Old Type ...	......	(100 volts)	(about 0·2)
15.	„ „ New Type...	......	(200 volts)	(about 10)
16.	Campbell Frequency-Teller.........	(500)	(0·2 amp.)	280
17.	Bell Telephone (double pole)	126	......	3000
18.	Ayrton and Perry Variable Inductance Standard.	10·6	(1 amp.)	20
19.	Standard Inductance Coil ($L=0·2$ henry).	10	(1 amp.)	46

As the numbers in the above Table throw an interesting light on the behaviour of many of the instruments it seems desirable to discuss them more fully in order.

(1) *Siemens Electrodynamometer.*—Measurements taken at the middle and the top of the swinging coil (of 4 turns) in direction perpendicular to the plane of the fixed coils gave $B=120$ and 40 respectively for the thin coil, and $B \doteqdot 21$ and 16 for the thick. It will be seen that when the thick coil is used the deflecting field is quite comparable with the earth's field.

This, of course, introduces an error with direct currents unless care is taken to place the instrument so that the direction of the earth's field is at right angles to that of the deflecting field, and in the proper sense, *i. e.* with the instrument looking east or west according to the direction of the current in the swinging coil. The above results show that the maximum variation at 15 amps. introduced by wrong placing (viz., due to a field equal to twice that of the earth, or 0·36) would be about 2·5 per cent. of the mean deflecting field. This was verified by placing the electrodynamometer on a well-levelled turntable, and connecting its thick coil with a quite steady source of continuous current by means of twisted flexible leads. In Table II. are shown the values of the current indicated by the instrument when turned into various positions, the first column giving the direction towards which the front was turned in each case.

TABLE II.

Direction.	Apparent Amperes.	Variation from Minimum.
N. . . .	15·00	0
W. . . .	15·20	1·3 °/₀
S. . . .	15·32	2·1 °/₀
E. . . .	15·20	1·3 °/₀

It will be seen that the extreme variation is 2·1 per cent., which agrees (within the limits of error of the instrument) with the 2·5 per cent. variation deduced from the observed fields. For lower currents the variation is much more, being in the inverse ratio of the current. With the thin coil the field due to the fixed coil is so much stronger that the variation is slight except at the very lowest currents.

The thick coil had 7 turns and an area of about 46 sq. centim., and the total maximum flux at full load was found to be 1850, giving mean $B \doteqdot 40$. At full load the power wasted is 6·2 watts and 9·3 watts for the thick and thin coils respectively.

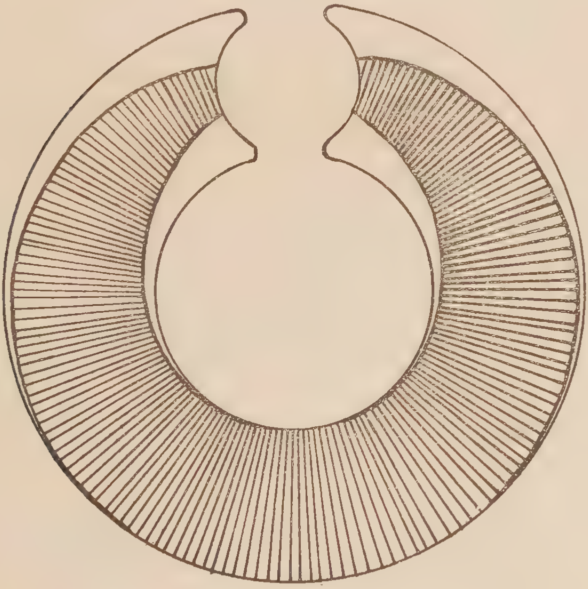
(2) *Kelvin Balance.*—The total flux through the central space of the coils was got by winding the search-coil round the supporting pillar, and taking throws by reversing the current. The resultant flux was about 1600 (for full load). By the astatic arrangement of the swinging coils the instrument is made independent of the earth's field. The self-inductance is about 0·0016 henry.

(3) *Bifilar Mirror Wattmeter.*—This instrument has ranges up to 50 or 100 amperes at 10 volts and upwards. The

numbers given refer to the fixed series-coil. With direct currents it is clear that precautions have to be taken to eliminate the effects of the earth's field.

(4) *D'Arsonval Galvanometer.*—This was a ballistic one (made by Paul) with a narrow swinging coil of the Ayrton-Mather type. The B given in the Table is that in the air-gap in the neighbourhood of the moving coil; it would seem to be sufficiently great to be practically unaffected by the magnetism of the earth. Besides, as it is an instrument for use in a fixed position, it is only the effect of variable external fields that need to be taken into account. This point will be discussed a little further on. To get an idea of how the flux in the steel varies from point to point along the annular magnet, an experiment was made with a ring-magnet of rectangular section, having an air-gap as shown in fig. 2.

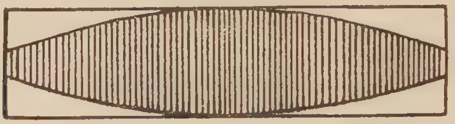
Fig. 2.



A small search-coil which could only just slip along the magnet was moved by jerks into successive positions, and the corresponding changes in the flux were calculated from the throws on a ballistic galvanometer in circuit with the coil.

Fig. 2 shows the result, the radial breadth of the shaded part being drawn proportional to the flux in the steel at each position. Fig. 3 is a similar diagram for an ordinary bar-magnet. In the ring-magnet the available air-gap flux was less than one third of the maximum flux.

Fig. 3.



(5) *Weston Voltmeter.*—It will be noticed that B in the air-gap of this instrument is very high, viz. 870. This might lead one to suppose that the earth's field would have no perceptible effect on its readings, but it must be remembered that the flux induced in a piece of iron or steel in the earth's field is usually very many times greater than the flux in air due to the earth alone. This can be easily shown by connecting a coil with a ballistic galvanometer and reversing the coil with regard to the earth's field first by itself and then with a soft iron core in it. The throws of the galvanometer will be enormously increased by the presence of the core. To find how far the earth's field affected the flux in a permanent magnet with a moderate air-gap, a coil was wound upon the circular one shown in fig. 2, and was connected with a galvanometer. The magnet was then turned round so as to quickly reverse the action of the earth's horizontal field upon it. The resulting throw on the galvanometer showed that the maximum B in the steel, which was about 5000, was only changed by 3 lines per sq. cm., *i. e.* by less than 0·1 per cent. The behaviour of the Weston magnet tallies with this, for when the instrument, with a steady voltage on its terminals, was turned round to face each point of the compass, no change in the reading could be detected, although the scale could be read to about 1 in 1000*.

(6) *Davies Voltmeter.*—In this instrument one side of the rectangular moving coil moves in a narrow cylindrical air-

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* At Professor Ayrton's suggestion I have re-tested the instrument at the higher readings, and have detected a variation of about 0·1 per cent.

gap between specially shaped pole-pieces of a strong permanent magnet. It gives a maximum angular deflexion of about 210°.

(7) *Evershed Ammeter.*—A coil of 6 turns magnetizes two small pieces of iron with a movable piece between them. B was measured at about the end of one of the fixed pieces where the movable piece faces it.

(8) *Ayrton and Perry Magnifying Spring Voltmeter.*—The small iron tube, which is surrounded by a coil and pulled down by it, is 7·2 centim. long, and the iron has a section of about 0·12 sq. centim. The number given in the table is the average B *in the iron* for the whole length of the tube for a load of 15 volts, *i. e.* 0·8 of fall load.

(9) *Richard Recording Ammeter.*—This has two solid iron cores (each 4 sq. centims. cross-section) round which the current is carried by a single turn of copper strip ; an iron armature carrying the pointer is attracted by these cores. B was measured between one core and the armature. It will be seen that the total flux is large ; thus a strong deflecting force is obtained which makes the friction of the recording pen of less account, but on the other hand much error from hysteresis comes in.

(10) *Dolivo Voltmeter.*—Here a thin wire of soft iron is drawn down into a solenoid. The value given is for the hollow core of the coil (including the wire).

(11) *Nalder Voltmeter.*—In this a small piece of soft iron moves in the magnetic field produced by a coil outside it. The number in the table is for the middle of the space within the coil.

(12). *Tangent Galvanometer.*—When the deflexion $=45°$, the resultant field $=$ earth's field $\div \cos 45°$

$$\doteqdot 0{\cdot}26 \text{ (in London).}$$

(13) *Kelvin Astatic Mirror Galvanometer.*—The galvanometer was made very sensitive and almost unstable by means of the controlling magnet. The mean control field (for this condition) was found by measuring its sensitivity and comparing it with that when the earth's field alone was used. The deflecting field for 1° would be less than 0·00002, and depends on the degree of astaticism of the suspended magnets.

(14) *Evershed Ohmmeter*—(Old type; polarized, astatic). The number given is a rough approximation to the B due to the shunt-coil alone at 100 volts. About the middle of the scale the field due to the series-coil would have the same value.

(15) *Evershed Ohmmeter*—(New type, with soft iron needle). The B given is that due to the shunt-coil alone (at 200 volts). It is clear that the earth's field cannot introduce much error. In any case the errors due to external fields can be eliminated (as the makers direct) by reversing the current and taking the mean of the readings.

(16) *Campbell Frequency-Teller.*—The B is measured between the vibrating strip and the attracting pole of the electromagnet. It will be seen that only a quite moderate field is required to throw the spring into strong vibration when it is adjusted to the right length for resonance.

(17) *Double Pole Bell Telephone.*—The diaphragm was so close to the poles of the permanent magnet as to form an almost closed magnetic circuit. A small search-coil was wound round one of the pole-pieces (area $=0{\cdot}24$ sq. centim.), and the diaphragm was then laid in its place. A throw of the galvanometer was got by pulling off both the diaphragm and the search-coil. Thus the flux-density given refers to the pole-pieces.

(18) *Ayrton and Perry Variable Standard of Self-Inductance.*—With the pointer at 0·038 henry, the total flux within the inner wooden bobbin was about 5200 for 1 ampere.

(19) *Self-Inductance Standard* ($L=0{\cdot}20$ henry).—This was a coil of 1158 turns of insulated copper wire (diameter 1·24 millim.), the outer diameter of the coil being 22 centim. and the height 9 centim. The B given in the table is the average for the whole cross-section of the coil at the middle of its height. The total flux corresponding to this was 17,500. B at the centre of this cross-section was 53.

In addition to the instruments already discussed, experiments were also made upon a number of meters of different types, some being for direct, and some for alternating currents. As some of the types vary in construction from year to year, a few words of description in each case will make the results clearer.

Aron Watt-hour Meter (1894 type).—Range to 50 amps. at 100 volts. Two pendulums, each carrying shunt-coils, are acted on by series-coils under them, one being accelerated and the other retarded. With both series and shunt-currents passing (at full load), the mean B between the fixed and movable coils was about 70.

Frager Watt-hour Meter.—Range to 10 amps. at 100 volts. A meter of the "Feeler" type, in which the shunt and series coils form an ordinary wattmeter, whose deflexions are integrated at intervals. The mean B was measured as near the centre of the shunt-coil as possible.

At full load . . . . B = 63,
With shunt-current alone . . B = 13,
With series-current alone . . B = 50.

Hookham Direct-Current Ampere-hour Meter.—Range to 100 amps. A small disk, surrounded by mercury which carries the current, is cut twice by part of the magnetic flux from a strong permanent magnet (of cross-section 7·5 sq. centim.). The disk is thus caused to turn. On the same spindle is a brake disk of copper (5·4 centim. diameter), which is also cut by a part of the flux from the same permanent magnet. Unfortunately it was not possible to take the meter to pieces, so the driving flux could not be measured. By slipping a search-coil along the permanent magnet, the total leakage was found to be over 26,000 lines. The total brake flux passes through the disk at four air-gaps, two and two in series, and has a value of about 5000 lines. The 8 pole-pieces which direct the flux have cross-sections of 1·53 centim. each. In two of them the iron near the air-gap is turned down so as to leave only a thin neck of about 0·12 sq. centim. cross-section. This is supposed to increase the permanence of the flux. At these necks more than $\frac{1}{4}$ of the total flux leaks from the iron. Whether they increase or diminish the permanence seems quite uncertain. The mean flux density at the four air-gaps was found to be about 1020.

The power spent in the meter at full load = 12·8 watts.

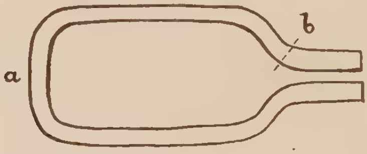
The power spent in turning the spindle (at 2·2 revolutions per second) with full load was measured by the method

(1) described below, and was found to be 0·016 watt*. Hence the efficiency of the meter *as a motor*=0·125 per cent.

Kelvin Ampere-hour Meter.—Range to 600 amperes. In this a thin iron core, kept highly magnetized by a shunt-current, is drawn down into a solenoid which carries the main current. The solenoid had 6 turns (*i. e.* 3600 ampere-turns at full load), and was about 16 centim. long. At full load the flux density at the lower end of the solenoid was over 250. With the shunt-current alone the total flux through the moving coil and core was about 330.

Elihu Thomson Watt-hour Meter.—Range to 50 amperes at 100 volts. This meter consists of a small ironless motor, in which the series-current goes through the field-magnet coils and the shunt-current through the armature. On the armature spindle is a brake disk of copper (13·5 centim. diameter), which passes between the narrow air-gaps of three permanent magnets of the shape shown in fig. 4. These magnets are often of different strengths, being chosen to give the proper brake-force for each individual meter. Their polar faces are about 7·5 sq. centim. The mean B in the air-gaps was about 700. By the method of placing the magnets the greater part of the flux acts on the brake disk. By slipping a search-coil along one of the magnets it was found that the total flux at *a* was about 15000, making B about 7000. Of this flux nearly one half remains in the steel as far as the section at *b*.

Fig. 4.



Driving Flux.—Without the shunt-current the full load current gives mean B $\doteqdot$ 130 along the axis of the series-coils. The shunt-current at 100 volts gives a field at right angles to this in which B = 10. The shunt-current also passes through a "compounding" coil fixed coaxially in one of the series-coils for the purpose of overcoming friction at the lower loads. The B due to this starting coil alone is about 3. There is a small stray field from the series-coils perceptible at the brake disk, but as it is less than the $\frac{1}{300}$ part of the field due to the permanent magnets its effect may be neglected.

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* After the author had measured the motor-efficiencies of several meters, Mr. Sidney Evershed somewhat anticipated him by announcing (Institution of Electrical Engineers, May 12th, 1898) one or two similar results, making no mention, however, of the method by which the results were obtained.

Effect of the Earth's Field.—Since the driving flux density at full load is only 130, it is clear that at the lower loads the rate of the meter may be considerably affected by the earth's field. To test this point the meter was levelled up on a turntable (as in the case of the electrodynamometer already described), and a constant load of 4·625 amperes at 100 volts was kept on it. The load was measured by a Kelvin balance and a reflecting multicellular voltmeter. The rate of the meter, *i. e.* spindle revolutions per watt-second, was then determined

(A) with alternating current;
(B) with direct current, the earth's field helping the driving field;
(C) the same, with the earth's field opposing the driving field.

Table III. shows the results of the tests.

TABLE III.

Position of Meter.	Current.	Rate.
(A) Facing East.	Alternating at 80~ per second.	0·0002125
(B) " West.	Direct.	0·0002135
(C) " East.	Direct.	0·0002212

It will be seen from (B) and (C) that by turning the meter round through 180° its rate at $\frac{1}{10}$ load is altered by 3·6 per cent., which is exactly what might have been predicted from the value of B given above. The rate with alternating current does not lie between the rates (B) and (C) as it ought, but is about 0·5 per cent. slower than (B).

Power Spent in Meter.—The resistance of the series-coil $\doteqdot$ 0·0066 ohm, and that of the shunt-coil $\doteqdot$ 2030 ohms; hence the power spent in heating the coils $=16{\cdot}5+4{\cdot}9=21{\cdot}4$ watts.

The power spent in actually driving the meter (at full load) was measured by two methods as follows :—

(1) An arm of about 10 centim. long was attached to the spindle of the meter (at right angles to it). With full load switched on, the tangential force f necessary to hold at rest the end of this arm was measured by the extension of a spiral spring which had been calibrated by known weights.

Then

$$\text{Power (in watts)} = 10^{-7} . f . 2\pi r . \frac{n}{t},$$

where

r = length of arm,

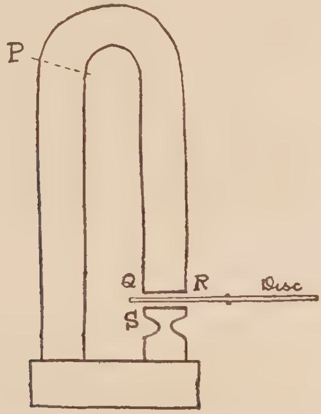
$\frac{n}{t}$ = number of revolutions per second when the spindle is free to move.

(2) In the second method the shunt-circuit was disconnected and joined directly to a sensitive galvanometer with a resistance of 12,000 ohms in circuit. A measured current was sent through the series-coils, and the spindle was turned at about the full rate. Thus the meter acted as a dynamo and gave a deflexion on the galvanometer. By watching the galvanometer-scale it was not hard to keep this deflexion steady, and from the known calibration of the galvanometer the voltage given by the armature was obtained. The total E.M.F. multiplied by the normal shunt-current (0·0493 amp.) gave the driving-power.

Method (1) gave for the driving-power at full load 0·020 watt, whilst method (2) gave 0·021 watt. Hence the efficiency of the meter as a motor is only 0·095 per cent.

Hookham Alternating-Current Watt-hour Meter.—Range to 10 amperes at 100 volts. In this meter a solid iron core forming an almost closed magnetic circuit is magnetized by a shunt-coil, which latter, by reason of its large inductance, carries a current which lags behind the main current by 50° to 60°. A smaller **U**-shaped piece magnetized by the main current has its poles close to the upper pole of the larger iron core, and one of the poles carries a copper screen. A small aluminium disk (8·8 centim. in diameter), partly between the poles, is turned by the rotary magnetic flux thus produced. The brake force acts on the same disk, and is due to a tall permanent magnet about 20 centim. high (with a narrow air-gap), as shown in fig. 5.

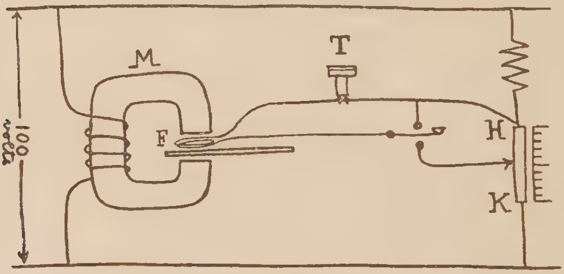
Fig. 5.



Brake Flux.—The maximum flux in this magnet was found to be near P, and had the value 19,820, corresponding to B=8800. Of this only 4400 lines ultimately cut the disk, much being lost by cross leakage from Q and S to the opposite limb. Thus less than $\frac{1}{4}$ of the maximum flux is made use of. The mean B between the poles was found to be 650.

Driving Flux.—The distribution of the somewhat complicated alternating field was traced by the following method, which also gave rough quantitative results. A telephone T (fig. 6) was arranged in a circuit with the search-coil F and the low-resistance strip H K in such a way that the strip could be switched out of circuit at will. One of the connexions to H F was through a sliding contact, so that the resistance of the part in the telephone-circuit could be varied from 0·05 ohm downwards. A current of 1 ampere was maintained in the strip and was from the alternating circuit which supplied the meter load. With the strip out of circuit the search-coil was moved into various positions, and the distribution of the alternating flux observed by means of the sound in the telephone. To measure the flux at any position H K was set to such a value that the small P.D. introduced by it into the telephone-circuit gave a sound of the same loudness as that given when the search-coil was placed in the flux. The absolute values given by this method came out 10 to 15 per cent. too small; but it proved a very convenient way of comparing the flux-densities at various positions. It was thus found that when the main current (full load) was switched on in addition to the shunt-current, the flux-density under the middle pole was nearly doubled. The most curious fact brought to light by the method was that across the air-gap of the permanent magnet a considerable alternating flux is induced, the value of B at that position being actually about $\frac{1}{4}$ of that between the poles of the shunt-magnet with the shunt-current alone. The permanent magnet forms a kind of secondary magnetic circuit directing around itself the alternating eddy-currents in the disk. Whether this has any sensible demagnetizing effect upon the permanent magnet the writer has not determined.

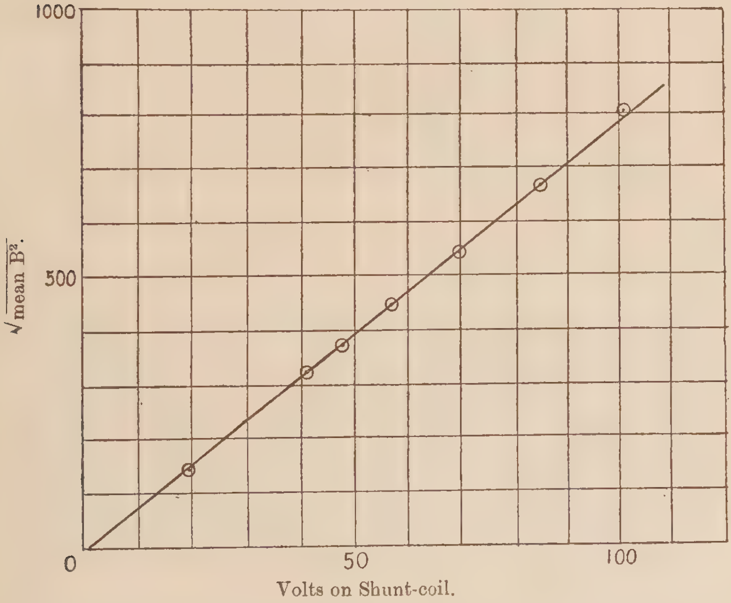
Fig. 6.



The fluxes were measured more exactly by the method of the search-coil and thermopile described above. With shunt alone the root of mean square B was about 50 in the air-gap and 800 in the iron core just above the shunt-bobbin. The shunt-current at 100 volts (86 ∼ per sec.) was found to be 0·031 ampere. If the current followed a sine curve its maximum value would be 0·044. When a direct current of this value was tried the fluxes produced were much larger than those with the (supposed) equivalent alternating current. This is partly due to the fact that the shunt-current does not follow the sine law, but is no doubt also due to the existence of eddy-currents in the iron core. That these currents even in the disk spread the flux and reduce the flux-density in the air-gap was shown qualitatively by placing a search-coil, connected with a telephone, in the air-gap of a ring electromagnet excited by alternating current. The sound in the telephone was lessened when a copper disk was held near the coil in the air-gap. The search-coil and thermopile method showed that the flux-density had been reduced by 8 per cent.

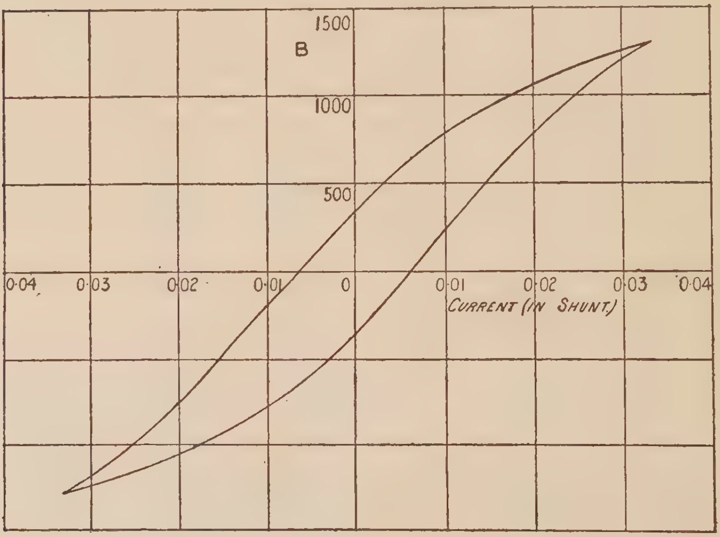
To find how the core-flux varied with alteration of the voltage on the shunt-coil, by the same method the B just above the shunt-bobbin was measured for a number of voltages from 20 up to 100 volts. It will be seen from the curve in fig. 7 that B is very nearly proportional to the potential-difference. In practice it is found that the speed of rotation is very nearly proportional to the voltage.

Fig. 7.



Power spent in Meter.—To approximate to the amount of power spent by reason of hysteresis when the shunt-current alone is on, the iron was carried through a cycle by means of direct current of such amount as to give a maximum B nearly corresponding to that given with alternating current at 100 volts. The curve obtained is shown in fig. 8.

Fig. 8.



From this curve it is not possible to get the exact value of the hysteresis loss, as the iron core is not a uniformly magnetized closed circuit. To get an idea, however, of the amount of the power wasted by hysteresis, let us suppose that the magnetic circuit is equivalent to a uniform iron ring of the same cross-section as the core of the shunt-coil, uniformly wound with the same number of turns n_1 as the shunt-coil * (carrying the same current), and traversed by a flux equal to that at the part of the meter-core for which the curve in fig. 6 was taken.

---

* The exact value of n_1 was not known, but the value (1200) used was estimated from the resistance and gauge of the shunt-wire and the size of the coil.

Let $c =$ current at any moment,
and $s =$ section of ring.
Then

Hysteresis-loss in joules per cycle

$$= \int c\, dv = 10^{-8} n_1 \int c\, d\Phi = 10^{-8} n_1 s \int c\, d\mathrm{B}$$
$$= 10^{-8} n_1 s \times (\text{area of curve})$$
$$= 10^{-8} \times 1200 \times 3{\cdot}7 \times \text{area}$$
$$\fallingdotseq 0{\cdot}00114.$$

$\therefore$ at $86\sim$ per second,

$$\text{Power spent} \fallingdotseq 86 \times 0{\cdot}00114 = 0{\cdot}098 \text{ watt.}$$

It was found by direct measurement that the actual power spent in the meter (with the shunt-current alone) was far larger than this. Accordingly measurements were made of the rise of temperature of the iron core by means of an iron-nickel thermopile. Three junctions of the pile were bound against the iron, which had its surface well insulated with paint. A pad of wadding was tied over the spot, and the thermopile was connected with a suitable galvanometer. The shunt-current was switched on for 120 seconds, the deflexion being read at intervals, and on breaking the current the cooling was observed for several minutes. The thermopile and galvanometer were calibrated with a known difference of temperature, and the curves of heating and cooling were drawn. The curve of heating was then corrected by means of the other curve, and thus the heating of the iron (corrected for cooling) was found.

At a spot just above the shunt-bobbin the corrected rise of temperature was 0°·87 C., whilst near the air-gap it was only about half of this. Taking account of this last fact, we may take that a volume of about 61 c.c. was raised in temperature by 0°·87 C.

Now

$$\text{Power spent} = 4{\cdot}2 \frac{\sigma . \mathrm{VD} . \partial\mathrm{T}}{t} \text{ watts,}$$

where σ = sp. heat of the iron,
V = volume „ „
D = density „ „
∂T = temperature-rise (corrected),
t = time in seconds;

therefore from the results above,

$$\text{Power} \doteqdot 1{\cdot}58 \text{ watts.}$$

Since the hysteresis-loss is about 0·10 watt, it will be seen that the eddy-current loss $\doteqdot$ 1·48 watts.

The power lost by eddy-currents in the disk was similarly measured, and was found to be about 0·02 watt.

In the shunt-coil the C^2R loss $=$ 0·57 watt; and hence the total copper and iron losses $=$ 2·17 watts.

Direct Measurement of Total Power.—The total power given to the meter was measured by the three-voltmeter method, in which was used a reflecting electrostatic voltmeter accurate to about 1 in 1000 at all the points of the scale required. The result obtained was 2·06 watts, which agrees fairly well with the total watts shown by the other methods.

Driving-Power.—The driving-power at full load was also measured by the method of the spring-balance described above; it was found to be 0·00073 watt. Taking account of the series-coil (whose resistance was 0·009 ohm) the total power spent at full load is over 3 watts; and hence

$$\text{Motor efficiency} \doteqdot 0{\cdot}024 \text{ per cent.}$$

Scheefer Alternating-Current Watt-hour Meter.—Range to 10 amperes at 200 volts. In this meter a pile of E-shaped iron stampings has the series-coil on one outer limb and the shunt-coil on the other. An aluminium cylinder is turned by the rotary field thus produced, and on the same spindle is a brake disk with a single magnet exactly like those in the Elihu Thomson meter.

The mean B between the poles of this magnet was about 480. With the shunt-current alone the $\sqrt{\text{mean } B^2}$ between the shunt-pole and the driving cylinder was 570. The total power spent with the shunt alone on is 11·5 watts, of which 7·1 watts are due to C^2R loss.

Shallenberger Alternating-Current Ampere-hour Meter.—Range to 20 amperes. A series-coil (of about 50 turns) and a small short-circuited secondary coil, with their axes at an angle of about 45°, produce a rotary field by which is turned a small disk with a soft iron rim. The brake-force is obtained by air-friction on four aluminium vanes.

By the search-coil and thermopile method it was found that inside the series-coil

$$\sqrt{\text{mean } B^2} \doteq 100.$$

Power spent in Meter.—The resistance of the series-coil was 0·025 ohm; hence the power spent in it = 10·0 watts.

The power spent in the copper stampings which form the secondary coil was found by measuring their rise of temperature with a small copper-iron junction. This rise (at full load) was found to be about 0°·33 C. per minute, the cooling being negligible. The volume of copper was about 21·4 c.c.; whence the power spent = 0·40 watt.

The driving-power was found to be 0·0069 watt; therefore the motor efficiency = 0·066 per cent.

Current in Secondary Coil.—The current in the short-circuited secondary coil could not be measured directly. Calculating from the dimensions of the coil, however, the resistance was found to be $8{\cdot}5 \times 10^{-6}$ ohm. From this and the value of the power (0·40 watt) we find that the secondary current attains the extraordinary value of 220 amperes.

For the sake of comparison some of the above results are collected in Table IV.

TABLE IV.

Name.	Driving B.	Brake B.	Power spent.	Motor Efficiency.
			watts.	per cent.
Elihu Thomson	130	700	21·4	0·095
Hookham (direct curr.) ...	(not measured)	1020	12·8	0·125
Hookham (alternat. curr.)	50	650	3·2	0·024
Shallenberger	100	......	10·4	0·066

In conclusion, it will be noticed that in motor meters the driving-flux density is of the order 100 and the brake B from 500 to 1000; also that the motor efficiencies are all very small, particularly in the case of the alternating-current meters. In all of them the greater part of the power taken is spent in heating conductors (either by eddy-currents or otherwise). If a small fraction of this wasted energy could be employed to overcome with certainty the friction at the lowest loads, a great advantage would be gained thereby.

June 7th, 1898.

DISCUSSION.

Prof. AYRTON said he wished to offer some remarks on behalf of Mr. Mather and himself. The paper would perhaps have received more adequate discussion at the Institution of Electrical Engineers, for it was chiefly of a technical character. The importance of neutralizing the effect of leads when using instruments with very weak fields, such as Siemens's electro-dynamometer, should be emphasized. In instruments of the Kelvin-balance type, where two opposed coils carry two opposed currents, the field spreads at the edges; the true "working" flux is not that directly between the coils. Mr. Campbell would have done better if he had used a long search-coil wound round one of the swinging coils, forming part of a vertical cylinder. It would have been well also to have supplied some experimental proof that the astatic arrangement of the swinging coils of the Kelvin balance makes the instrument independent of the earth's field. The effect of the earth's field is of the order 0·2, so that with instruments of the Weston type, with a field of the order 1000, it was sometimes assumed, erroneously, that the readings were practically independent of the earth's H. Prof. Ayrton's own tests showed that by turning a Weston voltmeter towards different points of the compass, the errors in a particular case were far greater than might be predicted from the above ratio; the induction in the voltmeter pole-space, due to the earth's field, was much higher than 0·2, the earth's field being exaggerated by the iron pole-pieces; it was not necessary to suppose that the magnetism of the permanent magnet caused the variation. The error observed was about 0·2 per cent. in a horizontal field, and 0·8 per cent. when the field of the voltmeter was parallel to the earth's induction. Here the induction in the gap was 1200, and $H=0·2$. In tests relating to the Ayrton and Perry magnifying-spring voltmeter, it was more important to know the B in the air-space *near* the iron than the B *within* the iron. Eddy currents might account for the extraordinary results obtained in the Shallenberger meter.

Mr. J. H. Reeves described a method he had adopted for measuring the effect of stray fields upon ammeters and voltmeters. The instrument to be tested is first mounted on a stand, and is brought under the influence of a large coil carrying a current. In this way fields of known magnitude can be superimposed on the working field throughout the range of the instrument, and the change of deflection due to them can be observed. From these known values the working field can be deduced. For let the current in the solenoid of the instrument at any moment be A amperes, producing a corresponding unknown working field of magnetic force X. Then X is proportional to the solenoid current, as measured by the indications of the instrument. If a magnetic force x is superimposed on X, then x is measured by x/X of A. If x is known, the working field X can be calculated from the change of deflection produced by the superposition. With Evershed ammeters the field measured in this way was in one instrument 200, and in another 226; or about one-third of Mr. Campbell's figure (700) for the Evershed ammmeter. Mr. Campbell's value of B did not represent the working field, but the field at the end of one of the fixed pieces of iron.

Mr. Campbell, in reply, said he thought the theory of electrical instruments to be well within the limits of Physics, and he had for that reason presented the paper to the Physical Society. The position chosen for the search-coil in the

Kelvin-balance tests may not have corresponded to the working flux, but it was near to the right position, and he had carefully specified the position chosen. His results as regards the Weston instrument differed from those of Prof. Ayrton; the errors he had observed for the particular voltmeter used were under 0·1 per cent. The earth's field probably produced an effect different for different Weston instruments, according to the degree of saturation of the permanent magnet. In his tests the Weston instrument did not have an iron case.

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